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NEW MODIFIED ANDERSON-DARLING AND CRAMER-VON MISES GOODNESS-OF-FIT TESTS FOR A NORMAL DISTRIBUTION WITH SPECIFIED PARAMETERS

THESIS

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The powers of the A-D tests modified by the median rank plotting and the mean rank plotting position are approximately 0.01% higher than the unmodified tests. The power of the C-VM test is 10% higher than the unmodified C-VM test for the Weibull, the lognormal, the exponential, and the double exponential distributions when the median rank plotting position was used. Using the mean rank plotting position also improves the power of the C-VM test for the uniform and the Weibull distributios; while it reduces the power for the exponential, the double exponential, and the lognormal distributions.

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NEW MODIFIED ANDERSON-DARLING AND CRAMER-VON MISES GOODNESS-OF-FIT TESTS FOR A NORMAL DISTRIBUTION WITH SPECIFIED PARAMETERS

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science (Operations Research)

Göksel Kahya, B.S.

First Lieutenant, TUAF

March, 1991

Preface

This thesis develops new, modified Anderson-Darling and Cramer-von Mises goodness-of-fit tests using the mean rank and median rank plotting positions for the normal distribution with specified parameters. The complete critical value tables are presented for each test. These tables can be used to test whether a set of observed values follows the normal distribution. Additionally, the power tables are presented for each distribution to help the readers understand the power relations clearly with different sample sizes.

First, I would like to extend my gratitude to my thesis advisor, Lt. Col. Donna Herge. Her guidance and perspective is deeply appreciated. I would also like to thank Dr. Joseph Cain for his suggestions and assistance.

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Göksel Kahya

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Abstract

This thesis improves the powers of the Anderson-Darling and Cramer-von Mises goodness-of-fit tests using the mean rank and the median rank plotting positions. A Monte Carlo simulation of 5000 repetitions is used to generate the critical values from $n = 4$ through $n = 80$ for the significance levels of $\alpha = 0.20$, $\alpha = 0.15$, $\alpha = 0.10$, $\alpha = 0.05$, and $\alpha = 0.01$. An extensive power study is performed to compare the powers of the modified tests to the known tests for the uniform, the exponential, the double exponential, the lognormal, and the Weibull distributions.

The powers of the A-D tests modified by the median rank plotting position and the mean rank plotting position are approximately 0.01% higher than the unmodified tests. The power of the C-VM test is 10% higher than the unmodified C-VM test for the Weibull, the lognormal, the exponential, and the double exponential distributions when the median rank plotting position is used. Using the mean rank plotting position also improves the power of the C-VM test for the uniform and the Weibull distributions; while it reduces the power for the exponential, the double exponential, and the lognormal distributions.

Finally, both A-D and C-VM tests modified by using the median rank plotting position have more power than the tests modified by using the mean rank plotting position.

NEW MODIFIED ANDERSON-DARLING AND CRAMER-VON MISES GOODNESS-OF-FIT TESTS FOR A NORMAL DISTRIBUTION WITH SPECIFIED PARAMETERS

I. Introduction

1.1 Background

The problems facing the Air Force continue to grow in size and complexity. Simulation modeling is one of the most commonly used techniques to resolve these problems (16:2). Decision makers evaluate the proposed solutions by using the results of the simulations, and in turn, are more able to plan future activities more efficiently.

Collection of data is one of the most important phases of the simulation process (16:43). After the collection of the data, analysts compare this data to a theoretical probability distribution. Then, the problem facing the analysts becomes how well the data fits the hypothesized distribution. The test to check if the hypothesized distribution fits the data is called a "goodness-of-fit" test. A goodness-of-fit test measures the degree of agreement between the distribution of an observed data sample and a theoretical statistical distribution (5:187). If such tests give a good fit, then the hypothesized distribution can be used in simulation modeling.

The most commonly used goodness-of-fit techniques are the Chi-square and the Kolmogorov-Smirnov (K-S) tests (5:168). Other goodness-of-fit techniques include Anderson-Darling (A-D) and Cramer-von Mises (C-VM) tests (6:109-114). These tests have been modified several times in order to get a better goodness-of-fit for the normal, uniform, Laplace, exponential, and Cauchy distributions using known and unknown parameters. The latest modification for the A-D and C-VM goodness-of-fit statistics was performed by Woodruff for the logistic distribution. He derived the critical value tables for the logistic distribution with unknown shape and location parameters. His power study indicated quite

good power against uniform and exponential alternatives. The modified A-D and C-VM tests had more power than the modified Kolmogorov-Smirnov test (19). However, not all distributions have been successfully examined for goodness- of-fit when the parameters of the distributions are unknown.

The normal or Gaussian distribution is the most important distribution in probability and statistics (16:32). Much of the data collected can be described by a normal distribution or can be said to be normal as a first approximation. After the data is collected, one can find that the data is often approximately normally distributed. Further, the assumption of the normal distribution as the basis for analyzing the sample data is often used in the development of advanced statistical theory. It can be concluded that the distributions of many sample statistics tend toward normality as the sample size increases.

The investigation for a better goodness-of-fit test which can be applied to the normal distribution still continues. No tests have been developed for the normal distribution with known parameters by using the new, modified Anderson-Darling and Cramer-von Mises statistics to determine a better goodness-of-fit. This test would be useful for the Air Force so that analysts can determine whether a random sample of data taken from an observed phenomenon behaves according to the normal distribution.

1.2 Problem Statement

How can the existing Anderson-Darling (A-D) and Cramer-von Mises (C-VM) tests be modified to produce a new, better goodness-of-fit test which can be applied to the normal distribution?

1.3 Research Objectives

The objectives of this thesis are to:

1. Generate and document the known A-D and C-VM critical value tables for the normal distribution.

2. Generate and document the A-D and C-VM critical value tables for the normal distribution with the modified A-D and C-VM statistical values. These tables will be used to test goodness-of-fit when the parameters of the distribution are known.
3. Conduct a power comparison between the modified and unmodified goodness-of-fit tests to determine which test can best detect a false normal distribution hypothesis.

1.4 Methodology

The research effort will consist of the following steps. The first step is to produce A-D and C-VM critical value tables for the normal distribution. The second step of the research is to produce the modified A-D and C-VM critical value tables for the normal distribution with the new modified A-D and C-VM statistical values. The final step will be to compare the powers of these two modified tests. For each step, computer subroutines will be written as required to perform any necessary calculations.

1.4.1 Critical Value Tables. During the first step, critical value tables will be generated with the known A-D and C-VM statistical values. Random deviates for a fixed sample of size n will be generated from a normal distribution using the Monte Carlo simulation. To evaluate the effect of sample sizes on the critical values, sample sizes of 4 through 80 will be used.

The estimated parameters will be used with the ordered normal deviates to calculate the values of the hypothesized distribution function at the order statistics. Based on the hypothesized and sample distributions, the A-D and C-VM statistics will be calculated using the appropriate equations. Each of these steps will be repeated 5000 times to generate 5000 independent A-D and C-VM statistical values. These critical values will be used to obtain the critical values of the tests.

The same procedure described in step one will be repeated in step two using the modified A-D and C-VM statistical values. Then, two complete sets of critical values will be presented for the modified and unmodified goodness-of-fit tests.

1.4.2 Power Comparison. Then the research will compare the powers of the modified A-D and C-VM tests to determine which tests can best detect a false normal distribution hypothesis. The power of a statistical test is the probability of correctly rejecting a false hypothesis. The null hypothesis that a set of sample deviates follows a normal distribution will be tested against the alternative hypothesis that the sample deviates follow some other distribution.

The A-D and C-VM test statistics will then be calculated under the null hypothesis that the random deviates follow the normal distribution with the specified parameters. The calculated statistics for each test will then be compared to the corresponding critical value tables obtained in step one and step two to determine whether to reject the null hypothesis. After repeating this 5000 times, the number of times each statistic exceeds the respective critical value will be counted for each sample size. This total will be the number of times the null hypothesis has been rejected. The power of the test for each alternative distribution will be the number of times the null hypothesis is rejected divided by 5000. This power comparison will be performed with some alternative distributions. These alternative distributions are the uniform, the exponential, the double exponential, the Weibull, and the lognormal distributions.

II. Literature Review

2.1 Introduction

A literature search has been completed on the goodness-of-fit techniques. Research studies published and unpublished have been investigated through the Current Index to Statistics, the Defense Technical Information Center [illegible] and previous AFIT theses efforts. The following paragraphs will review the literature pertinent to this research proposal. Specifically, the discussion covers the topics of hypothesis testing, test statistics, and power studies.

2.2 Discussion

From the earliest days of statistics, statisticians have begun their analysis by estimating a distribution for their observations, and then they have checked on whether or not this distribution is true. For years, a vast number of test procedures have appeared, and these procedures have come to be known as goodness-of-fit tests (6:1).

The goodness-of-fit test determines whether or not sample data could have been generated according to a particular distribution function. It achieves this by comparing the sample frequency distribution to the assumed theoretical frequency distribution (12:597).

Occasionally, assumptions are made regarding a distribution when there may be little prior statistical information. In these cases, it is more desirable that there be verification that the distribution fits the data. Comparisons of test data with an assumed distribution are accomplished by a goodness-of-fit test. Such a test evaluates how well the data fit the assumed distribution or determines if there is evidence of disagreement. The goodness-of-fit test is for disagreement rather than for agreement of the distribution with the data. The data are assumed to fit the distribution unless there is sufficient evidence to disprove the assumption (1:72).

One potential disadvantage of goodness-of-fit tests is that, even though an assumed distribution fits the data, there may be other distributions which fit the data equally well or perhaps even better. Another distribution may be of a different type or it may be

the same type of distribution but with different parameter values. Consequently, when applying a goodness-of-fit test, it is essential to remember that the test only rejects the assumed distribution when there is definite evidence that the distribution is incorrect; it does not mean that the selected distribution is the best one when there is not sufficient evidence to reject it (1:73).

2.2.1 Hypothesis Testing and Test Statistics. Hypothesis testing is the process of inferring from a sample whether to accept a certain statement about the population (5:76). Any statistical test of hypothesis works in exactly the same way and contains the following steps:

1. The hypotheses are stated in terms of the population.
2. A test statistic is chosen.
3. A rule is made, in terms of possible values of the test statistic, for deciding whether or not to accept or reject the null hypothesis.
4. On the basis of a random sample from the population, the test statistic is evaluated, and a decision is made to accept or reject the null hypothesis (14:429).

The hypothesis to be tested is called the "null hypothesis," and denoted by H_o. The alternative hypothesis, denoted by H_a, is the negation of the null hypothesis (5:77).

The functioning parts of a statistical test are the test statistic and an associated rejection region (5:78). The test statistic is a function of the sample measurements upon which the statistical decision will be based. If for a particular sample the computed value of the test statistic falls in the rejection region, the null hypothesis H_o is rejected and the alternative hypothesis H_a is accepted. If the value of the test statistic does not fall into the rejection region, then H_o is accepted (14:429).

There are two ways of making an incorrect decision in hypothesis testing. These are either rejecting a true null hypothesis or accepting a false null hypothesis. These two error types, type I and II respectively, have associated with them certain probabilities of the errors being made (5:79).

2.2.1.1 Chi-square Test Statistics. The oldest and best known goodness-of-fit test is the Chi-square test. Karl Pearson abandoned the assumption that biological populations are normally distributed, introducing the Pearson system of distributions to provide other models. The need to test fit arose naturally, and in 1900 Pearson invented his Chi-square test. Modern developments have increased the flexibility of Chi-square tests, especially when unknown parameters must be estimated in the hypothesized family (6:63). The Chi-square test compares observed frequencies with expected frequencies of the hypothesized distribution function. It is restricted to large samples, approximately 25 or greater (14:310). The major advantages of this test are that it can be applied to discrete populations, and it can be modified when parameter values are unknown by reducing the number of degrees of freedom; whereas the Kolmogorov-Smirnov test cannot be applied in these situations (13:68).

2.2.1.2 Statistics Based on the Empirical Distribution Function (EDF). A general class of statistics used for the goodness-of-fit test is called empirical distribution function statistics. The empirical distribution function is a step function, calculated from the sample, which estimates the probability distribution function. EDF statistics are measures of the discrepancy between the EDF and a given distribution function (6:97). For a given random sample of size n, Let $X_{(1)}, \ldots, X_{(n)}$ be the ordered statistics; suppose further that the distribution of X is $F(x)$. Then, the empirical distribution function is $F_n(x)$ defined by

$$F_n(x) = \frac{\text{Number of observations} \leq x}{n} \tag{2.1}$$

where

$$-\infty < x < \infty$$

More precisely, the definition becomes (6:98-99)

$$F_n(x) = 0, \qquad x < X_{(1)} \tag{2.2}$$

$$F_n(x) = \frac{i}{n}, \qquad X_{(i)} \leq x < X_{(i+1)}, i = 1, \ldots, n-1 \tag{2.3}$$

$$F_n(x) = 1, \qquad X_{(n)} \leq x \tag{2.4}$$

Thus $F_n(x)$ is a step function, calculated from the data; as x increases it takes a step up of height $\frac{1}{n}$ as each sample observation is reached.

The Kolmogorov-Smirnov statistic (K-S), which is one of the most well-known EDF statistics, is the largest vertical distance between the completely specified hypothesized cumulative distribution function (CDF) and the observed empirical distribution function (6:204). In some cases, the analyst may wish to specify completely an entire distribution and then test whether or not his sampled population has exactly that distribution in every detail. The K-S tests are appropriate in this situation, if the entire specification can be made prior to sampling (2:296).

There is a controversy between the K-S test and Chi-square test about which test is more powerful, but the general feeling seems to be that the K-S test is probably more powerful than the Chi-square test in most situations (2). D'Agostino and Stephens showed that EDF statistics were usually much more powerful than the Pearson Chi-square statistic (6:110).

Two other goodness-of-fit tests based on EDF statistics are Anderson-Darling (A-D) and Cramer-von Mises (C-VM) tests. The idea for the C-VM test is based on the squared integral of the difference between the EDF and the distribution being tested. The C-VM test statistic is defined as (6:101):

$$C - VM = \frac{1}{12\,n} + \sum_{i=1}^{n} \left[U_i - \frac{2i-1}{2n} \right]^2 \tag{2.5}$$

where,

n = sample size,

$U_i = F(X_{(i)})$ = CDF(for normal distribution).

and,

$$X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq \ldots \leq X_{(n)}$$

be n observations in order.

The C-VM test is also a function of the vertical distance between the hypothesized and the empirical distribution functions like the K-S test. But the difference is that the C-VM test not only considers the largest differences but also considers "n" differences between the two curves. Intuitively, the C-VM test statistic makes more complete use of the data than K-S statistic, but the facts fail either to prove or disprove such intuition (5:306). By using this fact, Conover stated that C-VM test seemed intuitively more appealing than the Kolmogorov test to some people (5:306).

Anderson-Darling brought the idea of incorporating a weight function into K-S and C-VM statistics (6:100). Primarily, the Anderson-Darling statistic is based on a weighted average of the squared discrepancy. Anderson-Darling test statistic is defined as (6:101):

$$A - D = -n - (1/n)\sum_{i=1}^{n}(2i-1)[\log U_{(i)} + \log(1 - U_{(n-i+1)})] \tag{2.6}$$

where,

n = sample size,

$U_i = F(X_{(i)})$ = CDF(for normal distribution),

and,

$$X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq \ldots \leq X_{(n)}$$

be n observations in order. In this expression, $\log(x)$ means $\log_e(x)$.

2.2.2 Power Studies. The power of a goodness-of-fit test is the probability that the test will reject the null hypothesis (14:473). The following discussion presents some of the modifications done for obtaining more powerful goodness-of-fit tests.

Green and Hegazy modified the A-D, C-VM, and the K-S goodness-of-fit tests where the parameters of the distribution were estimated. After the Monte Carlo power studies, they showed that the modified A-D and C-VM tests improved the power of the known tests (8:204).

Porter modified the A-D and C-VM tests for the Pareto distribution. He used the inverse transform technique to generate the random deviates. Then he used these deviates to

find the best linear unbiased estimates of the scale and location parameters. He concluded that the power of the test improved when the sample sizes increased. He also showed that A-D and C-VM tests were more powerful than the Chi-square, especially when the sample data were taken from the Weibull, the beta, or the normal distribution (15:7).

Ream modified K-S, A-D, and C-VM tests for the normal distribution to test the technique of reflecting data points about the mean and created tables of critical values of the modified K-S, A-D, and C-VM statistics. He used the technique developed by Efron (1979) which was a method for representing the order statistics on a continuous spectrum. He applied this test by plotting the values of the order statistics and represented the spaces between them as piecewise linear functions. He concluded that the powers calculated for symmetrical alternatives to the normal are asymptotically greater for the modified statistics than for the corresponding unmodified statistic values (17:62).

Bush modified the A-D and C-VM tests for the Weibull distribution. He presented a Monte Carlo method for obtaining the critical values of the modified A-D and C-VM goodness-of-fit tests for the three parameter Weibull distribution when the scale and location parameters were not specified. He also concluded that A-D and C-VM tests were not very powerful when the sample size was five. When the sample size increased, he showed that A-D and C-VM tests were more powerful than K-S test (3:52).

Viviano modified the A-D and C-VM tests for the Gamma distribution. He used the random deviates to estimate the maximum likelihood scale and location parameters. His power comparison showed that A-D goodness-of-fit test was more powerful against the lognormal distribution (18:47).

Stephens concluded after his power studies for normality and exponentiality that A-D statistic had better powers against some alternatives such as the uniform and the logistic distributions (6:167).

2.3 Conclusion

All these research studies have been done in order to provide more powerful tests for the certain distributions. They all modified the test techniques or used a new technique

for only one purpose, which is to obtain a "better" fit so that the analysts will be able to determine how well a set of data fits with some specified distribution.

Finally, this thesis effort is an investigation of a better goodness-of-fit test for the normal distribution with known parameters. Two modifications of both A-D and C-VM test statistics will be made to obtain the increased power. One modification is to replace the EDF in the A-D and C-VM test statistics with the median rank. The other modification replaces the EDF in the mean rank.

III. Methodology

3.1 Introduction

This chapter initially discusses the thought processes associated with this thesis, and then presents the specific procedures used in generating the critical values and the specific procedures used in the power tests. Specifically, the discussion covers the topics of Monte Carlo simulation method, calculation of the critical values, the powers of the goodness-of-fit tests, and finally the details of the computer programs.

3.2 Discussion

3.2.1 Monte Carlo Simulation Method. The Monte Carlo method is used for reliability prediction when an exact mathematical model cannot be developed economically or when it becomes too complex to permit timely evaluation. This method involves the determination of the distributions of the various elements in a system, selection of the random sample of each element, and combining of these samples to obtain a measure of the system performance or reliability. The process of random selection and determination of the system effects are repeated a large number of times and, each repetition results in another different estimate of the system characteristic that is being measured. The law of large numbers states that , as the sample size increases , the difference between the sample mean and the population mean becomes smaller, and the sample mean becomes an increasingly better estimate of μ. Then the distribution of the sample becomes a better representation of the distribution of the population (1:176).

An important characteristic of the Monte Carlo method is its reliance on computers to simulate random processes. The Monte Carlo approach is used in this thesis to generate the critical value tables for the A-D and C-VM goodness-of-fit techniques. The approach is to observe random variates chosen so that they directly simulate the random processes of the original problem. Ream tested the consistency of the critical values with different seeds each time using 150. 300, 500. 1000, and 5000 samples. The only number of samples that yielded consistent results through these computations at all levels of α was 5000 (17). This thesis uses 5000 samples in the computations as most of the studies have used in

this area. Another reason why only 5000 repetitions were used was to avoid using a large amount of computer time.

3.2.2 Modified Test Statistics. The computer was used to generate 5000 samples of size n. Each group of n normal deviates represents a simulated sample from the normal distribution. Then 5000 different values of test statistics were obtained by using the modified A-D and C-VM statistics. Two different modifications were used in this thesis:

1. Using the median rank plotting position
2. Using the mean rank plotting position

3.2.2.1 Using the Median Rank. There is a lot of discussion in the literature about the choice of the quantile probabilities. A frequently used formula is given by (6:164):

$$P_i = \frac{i - c}{n - 2c + 1} \tag{3.1}$$

where,

i = rank of the order statistics

n = total number of order statistics

c = some constant satisfying $o \leq c \leq 1$

This research will use $c = 0.3175$ since the resulting probabilities closely approximate medians of uniform $(0,1)$ order statistics. Using the c value chosen , the median rank value is given by:

$$P_i = \frac{i - 0.3175}{n + 0.365} \tag{3.2}$$

The median rank equation given above will be used for the further computations. Using the median rank formula given above, the modified A-D and C-VM statistics are given as:

$$The\ Modified\ C - VM = \frac{1}{12\, n} + \sum_{i=1}^{n} \left[U_i - \frac{i - 0.3175}{n + 0.365} \right]^2 \tag{3.3}$$

$$The\ Modified\ A-D = -n - \sum_{i=1}^{n}\left(\frac{2(i-0.3175)}{n+0.365}\right)[\log U_{(i)} + \log(1-U_{(n-i+1)})] \qquad (3.4)$$

The median rank plotting position seems to be more accurate than the other plotting positions (11:300).

The unmodified C-VM test statistic (2.5) is a function of the values of U_i which is the CDF value for the normal distribution function U_i is calculated at each X_i value in the random sample, and from this is subtracted the quantity $(2i-1)/2n$, which is the average of the empirical distribution functions just before and just after the jump that occurs at X_i , that is, the average of $(i-1)/n$ and i/n. That difference is squared so that positive differences do not cancel the negative differences, and the results are added together. After this brief explanation of C-VM test statistic, the main purpose of the research can be stated again as, how much more power will be provided with the new statistic using the median rank as the plotting position. The idea for the modified A-D goodness-of-fit test is again the same, which is inserting the median rank to the original A-D statistic (5:306).

3.2.2.2 Using the Mean Rank. The other plotting position used in this thesis is the mean rank. The c value in the quantile probability formula(3.1) is equal to 0 for the mean rank. The mean rank formula is given as:

$$Mean\ \ Rank = \frac{i}{n+1} \qquad (3.5)$$

Using the mean rank formula given above, the modified A-D and C-VM statistics are given as:

$$The\ \ Modified\ \ A-D = -n - \sum_{i=1}^{n}\left(\frac{i}{n+1}\right)[\log U_{(i)} + \log(1-U_{(n-i+1)})] \qquad (3.6)$$

$$The\ \ Modified\ \ C-VM = \frac{1}{12\,n} + \sum_{i=1}^{n}\left[U_i - \frac{i}{n+1}\right]^2 \qquad (3.7)$$

3.2.2.3 Plotting Positions. Given a series of observations ordered from smallest to largest (or vice versa), each event may be assigned a plotting position which is its cumulative probability. Recurrence intervals, which are reciprocals of the cumulative prob-

abilities, may also be used to define plotting positions (9:1615). The plotting positions are used to determine the percentiles of the distribution underlying a set of n ordered sample values (9:1615). The plotting position technique involves using a number of discrete values of the ordered test statistics and locating them on a continuous spectrum by representing the spaces between them as piecewise linear functions (9:1614). By linearly interpolating the desired percentiles between discrete values of the test statistics, more accurate critical values will be obtained (15:4.11).

The cumulative distribution function of a sample size of n is usually defined as a step function which jumps from $(i-1)/n$ to (i/n) at the i^{th} order statistic of the sample. When the plotting position i/n is used, the largest value cannot be plotted. If the plotting position $(i-1)/n$ is used, then the smallest value cannot be plotted. The probabilities 0 and 1 are off the scale for probability paper constructed for the normal distribution or for any other distribution unlimited in extent ($-\infty$ *to* ∞). Therefore statisticians have proposed different plotting positions, and they have tried to find the best plotting positions. Harter concluded that the optimum plotting position depended on the use that was to be made of the results and also depended on the underlying distribution (9:1615).

After his Monte Carlo study of plotting positions, Harter found that the median rank plotting position was one of the best plotting positions (10:320). The median rank plotting position, on which this thesis places the greatest emphasis, yields median unbiased estimates of x_i for a specified $F(x_i)$ and of $F(x_i)$ for a specified x_i (9:1625).

3.2.3 Calculation of the Critical Values. After calculating the 5000 statistics for each goodness-of-fit test, in order to determine the critical values, it is first necessary to determine the critical regions. This critical region is the set of all points in the sample space which result in the decision to reject the null hypothesis. After the critical region is determined , then the critical values can be found with a desired "level of significance", or α, which is the maximum probability of error in rejecting a true null hypothesis. If H_o is true, then the maximum probability of error in rejecting H_o is α; and therefore, the minimum probability of no error in accepting H_o is $1-\alpha$. In this case, the value of $1-\alpha$ represents a certain percentile of the 5000 ordered test statistic values.

The technique used in this thesis to determine the critical values is the bootstrap technique which was first used by Efron (7). Many statisticians have extensively used this technique to determine the critical values.

The research plots the n ordered test statistic values $x_1, x_2, \ldots, x_n$ along the horizontal axis and the n plotting position values $y_1, y_2, \ldots, y_n$ which are calculated with the median rank formula or mean rank formula along the vertical axis by using this technique. These values are assigned to positions 2 to $(n+1)$ on their respective values on the axes. There exists $[0,1]$ interval on the vertical axis performed by entering the endpoints $y_0 = 0$ at the position 1 and $y_{n+1} = 1$ at the position $n+2$. Letting the order statistics be represented by the horizontal axis and letting the vertical axis be scaled between zero and one, the statistics will be represented on a continuous function. Then, the corresponding endpoints on the horizontal axis are found by linear interpolation.

To plot the collection of n discrete values onto a fully continuous line between 0 and 1 requires an extrapolation of the plotting axes. The first point on the horizontal axis, x_0, is computed by linearly extrapolating from the second and third points, subject to a non-negativity restriction. Extrapolation is performed by using the standard linear slope-intercept formula $y = mx + b$ to compute the endpoints x_0 and x_{n+1}. To find the first endpoint on the horizontal axis, the slope is calculated by:

$$m = \frac{(y_{i+1} - y_i)}{(x_{i+1} - x_i)} \tag{3.8}$$

and the intercept is found by:

$$b = y_i - mx_i \tag{3.9}$$

Then the lower endpoint x_0 is found by:

$$C_p = (y_0 - b)/m = (0 - b)/m = -b/m \tag{3.10}$$

By using the nonnegativity rule, C_p is set to 0. So that:

$$x_0 = max(0, -b/m) \tag{3.11}$$

After both endpoints are calculated, n test statistic values and their two extrapolated endpoints are plotted on the $x - y$ axis. Then, the critical values corresponding to the desired percentiles are found by linearly interpolating between the points (x_i, y_i) and (x_{i+1}, y_{i+1}) using the formulas above; where C_p is the critical value for the desired percentile (15).

3.2.4 Power of the Goodness-of-fit Tests. The goodness of a test is measured by α and β, the probabilities of Type I and Type II errors mentioned in Chapter II, where α is chosen in advance and determines the location of the rejection region. Basically, the power of a test is the probability that the test will reject the null hypothesis.

Suppose that A is the test statistic and RR is the rejection region for a test of a given hypothesis concerning the value of a parameter θ. Then the power of the test, denoted by power (θ), is the probability that the test rejects H_o when the actual parameter value is θ. That is,

$$\text{power}(\theta) = \text{P (A in RR when the parameter value is } \theta)$$

The power of a test at $\theta = \theta_o$ is equal to the probability of rejecting H_o when H_o is true, where the null hypothesis is

$$H_o : \theta = \theta_o. \tag{3.12}$$

That is, $power(\theta_o) = \alpha$, the probability of a Type I error (14).

The main purpose of the power test is to test the thesis problem that the goodness-of-fit tests with the modified A-D and C-VM statistics will result in goodness-of-fit tests having higher powers than the powers of the ones with the unmodified A-D and C-VM statistic values.

To execute the power test , random deviates from some other alternative distributions of sample size n described below will be generated. Then the A-D and C-VM statistics

will be calculated with the modified A-D and C-VM statistic values. Since the critical values are known for the sample values of n and at the specified values of α , the A-D and C-VM statistics calculated will be compared to these respective values. This process will be repeated 5000 times. The number of the times each statistic value exceeds the respective critical value will be counted for each sample size of n. This total number will represent the number of the rejections of the null hypothesis. This total rejection number will be divided by 5000 to obtain the hypothesis rejection quotient. Then the power will be stated as:

$$The\ power\ at\ each\ \alpha - level = \frac{Total\ \ number\ \ of\ \ rejections}{5000} \tag{3.13}$$

After getting all the powers at each α-level, final conclusions will be derived for each alternative distribution.

The power study is performed at $n = 10$, $n = 20$, $n = 30$, $n = 40$, $n = 50$, $n = 60$, $n = 70$, and $n = 80$. By using the numbers shown above, it will be possible to compare the powers of the tests at different levels.

3.2.5 Specific Procedures. The research effort will consist of four phases. These phases will be performed by using the procedures and the techniques mentioned earlier. These phases are:

1. The A-D and C-VM critical value tables will be derived using the known A-D and C-VM statistic values.
2. The A-D and C-VM critical value tables will be generated using the modified A-D and C-VM statistic values.
3. The powers of the modified A-D and C-VM statistics will be determined by using alternative distributions. These alternative distributions are the uniform, the exponential, the double exponential, the Weibull, and the lognormal distributions.
4. The conclusions will be derived about the powers of the new modified statistics.

3.2.5.1 Generating the Critical Value Tables. During this phase, the critical value tables will be generated using the Monte Carlo simulation of random deviates. The following steps describe the procedure for generating the critical value tables.

1. Random variates will be generated for a given sample size of n from Normal distribution $N(0,1)$. To generate an array of ordered $N(0,1)$ random deviates.the subroutine named GGNO will be used from the International Mathematical and Statistics Library ($IMSL$).

2. The random variates generated will be standardized by using the following transformation:

$$Z_i = \frac{x_i - \bar{x}}{\sigma} \tag{3.14}$$

   These standardized data will be used in the further computations instead of the generated data.Then the test statistics will be computed from the Z_i values instead from the original random sample.

3. The hypothesized cumulative distribution function U_i will then be calculated for $i = 1, 2, \ldots, n$.

4. The A-D and C-VM statistics will be calculated using the known and the modified statistics.

5. Each of these four steps described above will be repeated 5000 times to generate 5000 independent statistics.

6. These 5000 statistics will be ordered from smallest to largest. Using the plotting position technique, the $80^{th}, 85^{th}, 90^{th}, 95^{th}$,and 99^{th} percentiles of the distributions of the A-D and C-VM statistics will be determined by linear interpolations. These percentiles represent the .20, .15, .10, .05, .01 levels of significance. Then the critical value tables will be documented. These critical value tables will be fully documented to give a clear motivation for the future studies.

The flowchart representation for generating the critical values is shown on the next page.

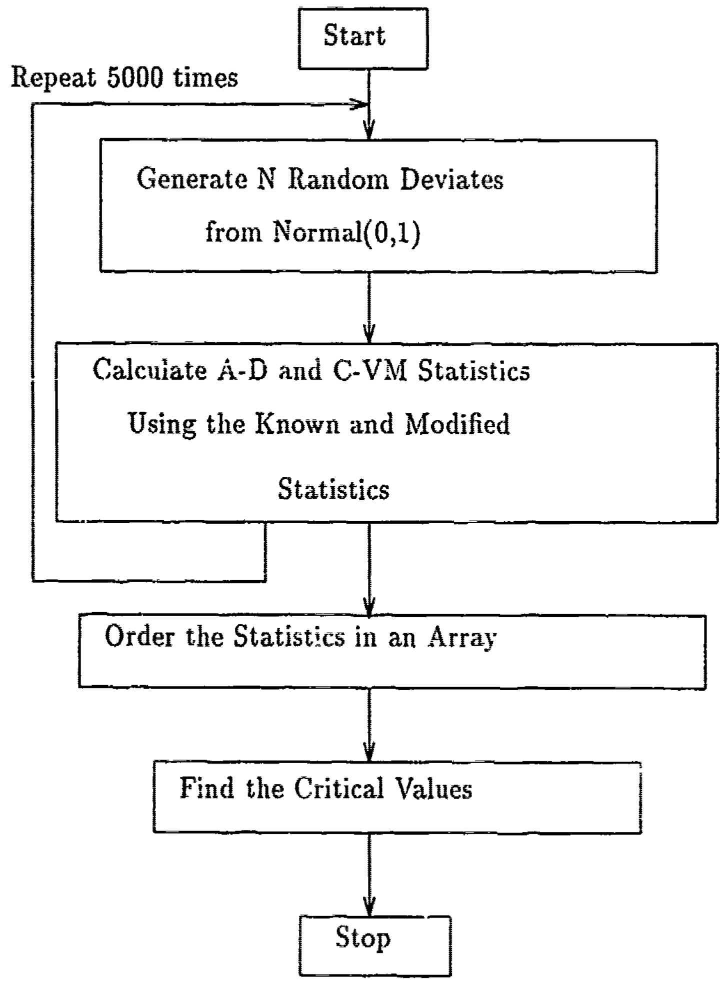

Figure 3.1. Generation of Critical Values

3.2.5.2 Power Comparison. In this stage of the research effort, the powers of the modified A-D and C-VM statistics will be calculated. As mentioned earlier, the power of a test is the probability that the test will reject the null hypothesis. The null hypothesis is that a sample of deviates follows a specified distribution where the alternative hypothesis is that the sample of deviates follows some other distribution.

H_o : *Sample deviates follow a specified distribution.*

H_a : *They follow some other distribution.*

The steps for this phase of the research are as follows:

1. Random deviates for the selected distributions will be generated.
2. Under the given hypothesis test given above, the test statistics will be compared to the corresponding critical values in the table for a given level of significance α . If the test statistic is equal to or greater than the critical value in the table, then the null hypothesis (H_o) is rejected.
3. The first two steps will be repeated 5000 times, and these rejections will be counted to obtain the power of each statistic value.

The flowchart representation for the power tests is shown on the next page.

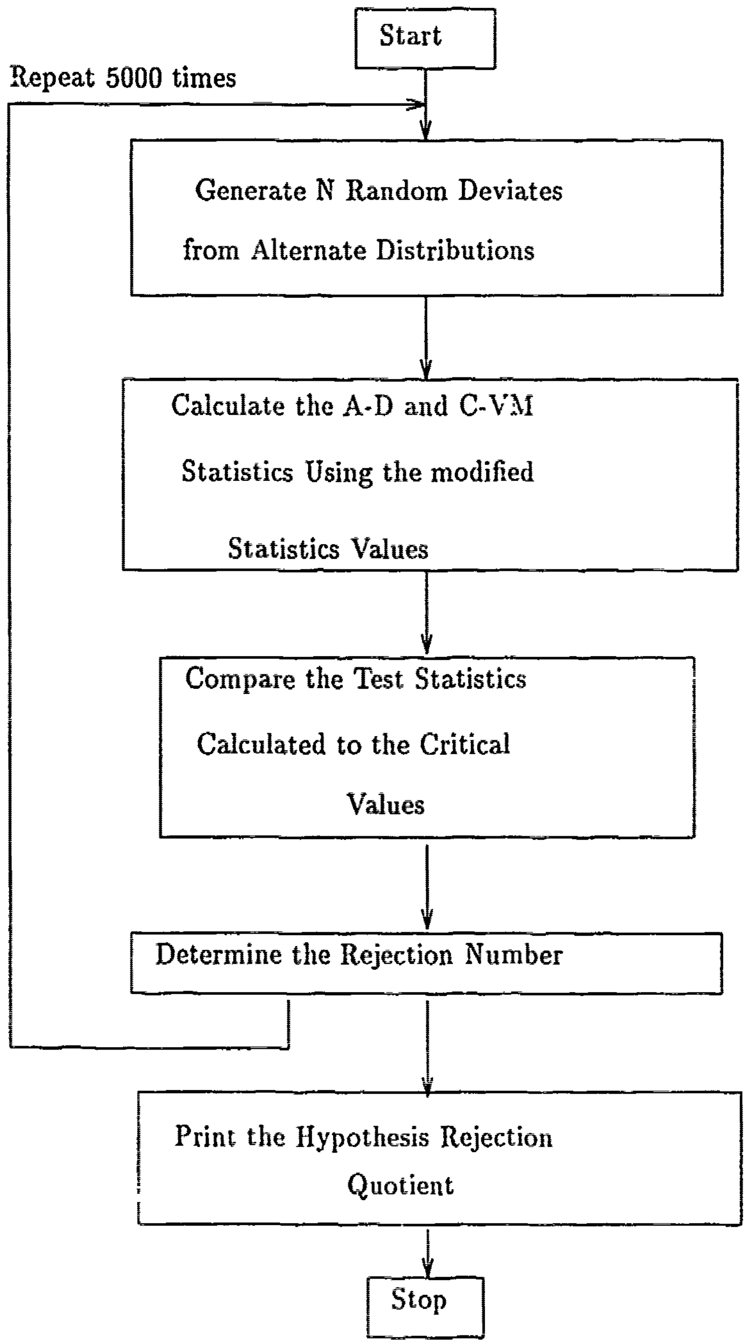

Figure 3.2. Power Comparison

3.3 Conclusion

This chapter represented the specific procedures used in this research study. First the discussion about the Monte Carlo simulation method was explained and then the techniques to generate the critical values were presented with the modified A-D and C-VM statistics.

The next step will be generating the critical values and performing the power tests using the procedures explained in this chapter.

IV. Power Studies

4.1 Introduction

This chapter presents the critical value tables and the power comparison tables. The methodology described in the previous chapter is used to perform the power comparisons. The complete tables of the critical values are presented. Specifically, the discussion covers the topics of the critical value tables and the power comparison tables.

4.2 Discussion

4.2.1 Critical Value Tables. The complete tables of critical tables are presented for the following tests for the sample sizes $n = 4$ through $n = 80$:

1. The Anderson-Darling test
2. The Anderson-Darling test modified by using the median rank
3. The Cramer-von Mises test
4. The Cramer-von Mises test modified by using the median rank

The critical values for the A-D and C-VM tests are generated by using the known A-D and C-VM statistics. The critical values for the modified A-D and CV-M tests are generated by using the new modified statistics given by Equations (3.3). (3.4). (3.6). and(3.7). These four critical value tables are included in the study for clear comparisons between the tests.

Table 4.1. Critical Values for the Anderson-Darling Test (A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
4	0.3963525	0.4282169	0.4779935	0.5479218	0.6871245
5	0.4273410	0.4686368	0.5215520	0.6037830	0.8042023
6	0.4363447	0.4730742	0.5266748	0.6195503	0.8272897
7	0.4451900	0.4847007	0.5462756	0.6435997	0.8582927
8	0.4602733	0.5014591	0.5624942	0.6760206	0.9111053
9	0.4656225	0.5135141	0.5725465	0.6861147	0.9495367
10	0.4768448	0.5238467	0.5830092	0.6825218	0.9336353
11	0.4752173	0.5213203	0.5879040	0.6977583	0.9280258
12	0.4811802	0.5301452	0.5988379	0.7005739	0.9557297
13	0.4799352	0.5256196	0.5938590	0.7120132	0.9731674
14	0.4804397	0.5274926	0.5921049	0.7081370	0.9824843
15	0.4835605	0.5303540	0.5964639	0.6977839	0.9436393
16	0.4897432	0.5299063	0.5925674	0.7135734	0.9995052
17	0.4893551	0.5400362	0.6148014	0.7455606	0.9911392
18	0.4801931	0.5276967	0.5963792	0.7095528	0.9354473
19	0.4942989	0.5429153	0.6165494	0.7317066	0.9762077
20	0.4821519	0.5319233	0.6044996	0.7169857	1.000139
21	0.4883166	0.5376720	0.6116218	0.7228747	0.9610729

Table 4.2. Continued (A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
22	0.4930449	0.5406342	0.6087568	0.7233239	1.004090
23	0.4965525	0.5448771	0.6276847	0.7518797	1.024874
24	0.4945974	0.5435820	0.6202430	0.7262725	0.9730281
25	0.4968777	0.5474281	0.6153478	0.7253182	0.9874505
26	0.4870243	0.5349074	0.6020432	0.7138987	0.9905736
27	0.5064116	0.5555526	0.6255779	0.7485665	1.035611
28	0.5035725	0.5482607	0.6243295	0.7442713	0.9996597
29	0.4967098	0.5470409	0.6043720	0.7331629	1.018337
30	0.5002642	0.5460176	0.6153994	0.7396423	1.019706
31	0.4920015	0.5412827	0.6096362	0.7206756	0.9634513
32	0.4977684	0.5414925	0.6100102	0.7216454	1.037556
33	0.4967517	0.5462971	0.6123351	0.7156105	0.9811327
34	0.4958267	0.5410481	0.6100807	0.7225074	1.010662
35	0.4973602	0.5470352	0.6169090	0.7334061	0.9903298
36	0.5038891	0.5516129	0.6242867	0.7248192	0.9966785
37	0.4977741	0.5424100	0.6118068	0.7373582	1.014974
38	0.4951401	0.5434284	0.6065102	0.7302265	0.9693126
39	0.4948826	0.5474167	0.6189229	0.7319393	1.035174

Table 4.3. Continued (A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
40	0.5036736	0.5492897	0.6088142	0.7162741	0.9774227
41	0.4935474	0.5456238	0.6108913	0.7262822	0.9670739
42	0.4987202	0.5480480	0.6173229	0.7275543	1.032360
43	0.5078812	0.5625439	0.6357059	0.7474669	1.015957
44	0.5057907	0.5523701	0.6249388	0.7439404	1.030191
45	0.5075741	0.5600471	0.6272904	0.7511843	1.049332
46	0.5009937	0.5491180	0.6143340	0.7353191	1.079781
47	0.4968414	0.5439053	0.6071224	0.7375965	1.043659
48	0.5051956	0.5512581	0.6226959	0.7505187	1.048113
49	0.4975300	0.5474415	0.6134128	0.7330531	0.9863949
50	0.5064582	0.5549145	0.6306038	0.7509441	1.017109
51	0.5016975	0.5513611	0.6221276	0.7502614	1.011376
52	0.5054054	0.5616665	0.6354789	0.7535743	1.013035
53	0.4991112	0.5465317	0.6146659	0.7347736	0.9884982
54	0.4980010	0.5479546	0.6144963	0.7294196	0.9858552
55	0.5054150	0.5574017	0.6312294	0.7499961	1.044233
56	0.5054855	0.5514259	0.6198158	0.7405357	1.001810
57	0.5003051	0.5516874	0.6244812	0.7361297	1.019163

Table 4.4. Continued (A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
58	0.5041008	0.5561218	0.6305560	0.7591591	1.017233
59	0.4955597	0.5473194	0.6220034	0.7387219	0.9823399
60	0.5019493	0.5448074	0.6112021	0.7257367	1.023222
61	0.4970607	0.5500755	0.6146068	0.7381287	1.029202
62	0.5038795	0.5488949	0.6174755	0.7389317	0.9899883
63	0.5008945	0.5522652	0.6225338	0.7408447	0.9984367
64	0.5083809	0.5651475	0.6351047	0.7575722	1.035576
65	0.5034600	0.5512009	0.6210823	0.7369995	1.019421
66	0.5056152	0.5513268	0.6228599	0.7395401	0.9776191
67	0.4956703	0.5476418	0.6135138	0.7234917	1.002739
68	0.4987144	0.5423241	0.6143417	0.7315444	0.9953613
69	0.4891967	0.5374489	0.6148720	0.7375908	1.016052
70	0.5006447	0.5449486	0.6086006	0.7174299	0.9944189
71	0.5036469	0.5571595	0.6310691	0.7516361	0.9956664
72	0.5042152	0.5534744	0.6187972	0.7528877	1.098568
73	0.4993744	0.5456963	0.6249349	0.7475891	0.9832656
74	0.4938010	0.5401573	0.6125678	0.7294769	1.027989
75	0.5070686	0.5569229	0.6184120	0.7290077	1.064887

Table 4.5. Continued (A-D)

	Level of Significance (α=Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
76	0.5072517	0.5548745	0.6221275	0.7382354	0.9811439
77	0.5040436	0.5481758	0.6155281	0.7394637	1.029260
78	0.5019684	0.5506668	0.6074256	0.7241516	1.010888
79	0.4939728	0.5408135	0.6081811	0.7273674	1.041717
80	0.5018578	0.5535432	0.6213608	0.7532310	1.046329

Table 4.6. Critical Values for the Modified Anderson-Darling Test (Mod. A-D)

	Level of Significance (α=Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
4	1.582756	1.617663	1.671845	1.748723	1.901822
5	1.596477	1.640916	1.698639	1.786559	2.001122
6	1.592883	1.631737	1.689737	1.786873	2.006685
7	1.593557	1.634419	1.700119	1.801773	2.026566
8	1.602466	1.645357	1.709502	1.827742	2.073187
9	1.601761	1.651268	1.713816	1.832235	2.107482
10	1.609766	1.657253	1.719365	1.821748	2.080344
11	1.603089	1.651914	1.720261	1.835227	2.068199
12	1.607941	1.657570	1.728033	1.832728	2.093671
13	1.603700	1.650733	1.720114	1.841662	2.111019
14	1.602948	1.650619	1.716714	1.834993	2.112786
15	1.603284	1.650743	1.718761	1.821638	2.075947
16	1.608843	1.650144	1.712707	1.837376	2.129101
17	1.606355	1.658886	1.735003	1.866905	2.117235
18	1.595449	1.644476	1.714828	1.829279	2.056677
19	1.609162	1.659450	1.733142	1.848537	2.097478
20	1.595428	1.647300	1.720243	1.836084	2.122495
21	1.601188	1.650129	1.726903	1.838660	2.080507

Table 4.7. Continued (Mod. A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
22	1.605027	1.654015	1.723168	1.837928	2.123885
23	1.607399	1.656692	1.740698	1.866060	2.144170
24	1.604913	1.654651	1.731568	1.840148	2.089002
25	1.606775	1.657915	1.725787	1.838591	2.104431
26	1.596288	1.645237	1.714891	1.824636	2.104944
27	1.614830	1.665790	1.735625	1.862446	2.148955
28	1.612313	1.657729	1.735475	1.854534	2.111488
29	1.604225	1.655478	1.714090	1.842946	2.132516
30	1.607267	1.654267	1.724511	1.850140	2.132860
31	1.598879	1.648886	1.717329	1.830383	2.074648
32	1.604487	1.650028	1.717541	1.829722	2.149841
33	1.602903	1.654169	1.719116	1.824087	2.091438
34	1.601793	1.648222	1.716944	1.830750	2.120502
35	1.602797	1.653866	1.723196	1.843155	2.100450
36	1.608988	1.657530	1.729572	1.832800	2.105381
37	1.602711	1.648737	1.717335	1.846005	2.122217
38	1.599356	1.648787	1.712374	1.836834	2.077459
39	1.599217	1.651529	1.724192	1.837242	2.143903

Table 4.8. Continued (Mod. A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
40	1.608160	1.654381	1.713497	1.822373	2.084799
41	1.597586	1.649262	1.714718	1.832047	2.075401
42	1.602928	1.652935	1.722067	1.833937	2.139005
43	1.611481	1.666656	1.740896	1.854076	2.122475
44	1.608974	1.657139	1.729172	1.848476	2.138180
45	1.610071	1.663834	1.731770	1.854791	2.154166
46	1.603903	1.652592	1.718451	1.838369	2.184988
47	1.604656	1.654627	1.728684	1.855408	2.123127
48	1.602488	1.654079	1.726776	1.846361	2.114164
49	1.597328	1.644581	1.706415	1.833174	2.111530
50	1.606836	1.661249	1.734949	1.861311	2.148796
51	1.609284	1.659309	1.738390	1.839491	2.131019
52	1.603733	1.654411	1.722675	1.845199	2.132654
53	1.603451	1.652371	1.713387	1.824596	2.170734
54	1.598221	1.649824	1.720283	1.833208	2.110764
55	1.604191	1.658361	1.733399	1.844929	2.126796
56	1.600681	1.653149	1.724302	1.841839	2.134912
57	1.604868	1.653820	1.726812	1.835598	2.124434

Table 4.9. Continued (Mod. A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
58	1.608841	1.658838	1.720072	1.842489	2.122024
59	1.601927	1.653996	1.727892	1.841658	2.123772
60	1.604246	1.655979	1.722711	1.850096	2.099027
61	1.601652	1.649261	1.721050	1.842003	2.114895
62	1.609081	1.664414	1.726728	1.855917	2.137005
63	1.606510	1.658436	1.731819	1.851661	2.188877
64	1.595528	1.643341	1.708275	1.821423	2.082424
65	1.614925	1.665764	1.738296	1.848522	2.120224
66	1.603245	1.655537	1.723713	1.853168	2.155743
67	1.605503	1.652283	1.727871	1.854118	2.106010
68	1.601856	1.646454	1.719967	1.821701	2.072914
69	1.602928	1.650131	1.721455	1.826721	2.068020
70	1.601944	1.646496	1.711170	1.825813	2.108289
71	1.598667	1.644791	1.713329	1.848534	2.133702
72	1.604134	1.651684	1.718487	1.842300	2.086609
73	1.593395	1.643261	1.709183	1.825314	2.140033
74	1.602291	1.652550	1.724125	1.857033	2.137829
75	1.594906	1.645405	1.721660	1.841076	2.113717

Table 4.10. Continued (Mod. A-D)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
76	1.608738	1.661160	1.740814	1.863881	2.192662
77	1.599689	1.650547	1.725956	1.831131	2.113838
78	1.598564	1.648224	1.718864	1.836502	2.126775
79	1.600918	1.649132	1.719360	1.825165	2.155197
80	1.602745	1.650009	1.714119	1.824196	2.109204

Table 4.11. Critical Values for the Cramer-von Mises Test (C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
4	0.068717889	0.075482644	0.085986301	0.1027864	0.1329755
5	0.074497014	0.81921719	0.092780255	0.1088244	0.1519448
6	0.073810749	0.081494279	0.092883252	0.1105654	0.1521890
7	0.075224698	0.082806356	0.094552420	0.1141220	0.1578436
8	0.077101640	0.085106254	0.097887017	0.1198000	0.1676036
9	0.078383930	0.086605310	0.098667294	0.1204440	0.1719634
10	0.079117730	0.087914445	0.099269629	0.1196247	0.1692951
11	0.078922927	0.087871805	0.1000359	0.1211092	0.1659990
12	0.09329304	0.088333301	0.1021110	0.1221171	0.1716552
13	0.078199118	0.088464923	0.1005328	0.1214648	0.1737344
14	0.078987874	0.087575160	0.099740401	0.1212310	0.1704127
15	0.079056472	0.087549478	0.099057473	0.1191476	0.1690551
16	0.080250062	0.088360421	0.1004809	0.1211501	0.1750606
17	0.079889290	0.090032421	0.1039800	0.1289154	0.1754416
18	0.078992881	0.086529292	0.099361897	0.1195607	0.1663492
19	0.081109054	0.089803994	0.1024809	0.1242585	0.1726213
20	0.078324966	0.087843895	0.1009991	0.1217474	0.1816628
21	0.079958476	0.089077838	0.1013554	0.1227503	0.1673577

Table 4.12. Continued (C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
22	0.080977112	0.088679813	0.1018423	0.1238366	0.1773855
23	0.080085248	0.090274885	0.1034425	0.1300206	0.1784900
24	0.07939031	0.089777172	0.1020681	0.1217616	0.1747164
25	0.080698572	0.090523042	0.1030444	0.1230224	0.1699734
26	0.078952312	0.087606333	0.099764936	0.1190569	0.1706200
27	0.081380449	0.090854980	0.1030964	0.1283479	0.1768174
28	0.081017911	0.089316107	0.1036277	0.1256568	0.1728324
29	0.080269307	0.089592382	0.1017790	0.1247838	0.1799154
30	0.080438256	0.089790776	0.1020064	0.1237940	0.1782377
31	0.079447605	0.088063426	0.1010344	0.1231003	0.1714168
32	0.079861253	0.089245029	0.1006432	0.1208329	0.1808827
33	0.080136381	0.089789651	0.1020840	0.1211182	0.1760548
34	0.079988673	0.088616826	0.1018256	0.1229455	0.1763405
35	0.08059419[illegible]	0.089403555	0.1023602	0.1261002	0.1755391
36	0.081034966	0.089757077	0.1015147	0.1221960	0.1723452
37	0.081238300	0.088818684	0.1010038	0.1264580	0.1787453
38	0.080053426	0.088507704	0.1006704	0.1224907	0.1633722
39	0.080003247	0.088844657	0.1019481	0.1246798	0.1788400

Table 4.13. Continued (C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
40	0.080477841	0.089769222	0.1011164	0.1196890	0.1695593
41	0.080171429	0.088017531	0.1005671	0.1244502	0.1731949
42	0.080914006	0.089308478	0.1018946	0.1228404	0.1791724
43	0.081907399	0.091332987	0.1055718	0.1283962	0.1750055
44	0.081682011	0.091009520	0.1021398	0.1265333	0.1841971
45	0.081103258	0.090990670	0.1033571	0.1269290	0.1839232
46	0.080394760	0.089128010	0.1025386	0.1233494	0.1827025
47	0.079413734	0.088695556	0.1002844	0.1236093	0.1751411
48	0.080372520	0.090312369	0.1031824	0.1251452	0.1786935
49	0.080017462	0.089598760	0.1016602	0.1224438	0.1748529
50	0.080518857	0.089651220	0.1030667	0.1253582	0.1759703
51	0.080380484	0.089901276	0.1032750	0.1250169	0.1754528
52	0.080479883	0.090384185	0.1038498	0.1272965	0.1822425
53	0.079644896	0.088755876	0.1016509	0.1230755	0.1731220
54	0.079933308	0.088640191	0.1029210	0.1227959	0.1692909
55	0.081148617	0.090576157	0.1043266	0.1260005	0.1849225
56	0.080681525	0.090332493	0.1028001	0.1235662	0.1738220
57	0.080314606	0.089454904	0.1027690	0.1240410	0.1762333

Table 4.14. Continued (C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
58	0.080621526	0.090482786	0.1028826	0.1240699	0.1786074
59	0.079781540	0.088063873	0.1007847	0.1235907	0.1725108
60	0.079344243	0.088021867	0.1004375	0.1237136	0.1681552
61	0.079368405	0.088960953	0.1016442	0.1216617	0.1700009
62	0.080933124	0.090191275	0.1037772	0.1250453	0.1755820
63	0.079995684	0.089590319	0.1018710	0.1251102	0.1737180
64	0.080818497	0.090112224	0.1029750	0.1255121	0.1772580
65	0.081838332	0.091066293	0.1039007	0.1274379	0.1774001
66	0.081019178	0.090116628	0.1044304	0.1262270	0.1695016
67	0.078763321	0.087924048	0.1029343	0.1247742	0.1802715
68	0.081587173	0.091857091	0.1043757	0.1296844	0.1813279
69	0.082457997	0.091501348	0.1051687	0.1271503	0.1730216
70	0.081171773	0.090048738	0.1042446	0.1254718	0.1781813
71	0.080247104	0.090262674	0.1030740	0.1260096	0.1740322
72	0.080701694	0.089826591	0.1023556	0.1250563	0.1802059
73	0.080590956	0.090665735	0.1037948	0.1263535	0.1768728
74	0.081318602	0.091510229	0.1035885	0.1236244	0.1736130
75	0.081097774	0.090924881	0.1030945	0.1260590	0.1815204

Table 4.15. Continued (C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
76	0.081169255	0.090258129	0.1024938	0.1219029	0.1646494
77	0.081446439	0.090075724	0.1035862	0.1257362	0.1830947
78	0.081172861	0.090091988	0.1031678	0.1263537	0.1774466
79	0.080688916	0.089598104	0.1008036	0.1215363	0.1733790
80	0.079321444	0.089231446	0.1024222	0.1257416	0.1837831

Table 4.16. Critical Values for the Modified Cramer-von Mises Test (Mod. C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
4	0.1171122	0.1258052	0.1399408	0.1631716	0.2042842
5	0.1141850	0.1239169	0.1363130	0.1598520	0.2155906
6	0.1068020	0.1162525	0.1293700	0.1531270	0.2002948
7	0.1032833	0.1128287	0.1268466	0.1502074	0.1996575
8	0.1016296	0.1121755	0.1274291	0.1523345	0.2109509
9	0.1002520	0.1098706	0.1242078	0.1462136	0.2049321
10	0.096721902	0.1067272	0.1205297	0.1434972	0.2080106
11	0.094881043	0.1041535	0.1188708	0.1445517	0.2029956
12	0.094152793	0.1041171	0.1179808	0.1418343	0.2027317
13	0.094214745	0.1040925	0.1186687	0.1422543	0.1956540
14	0.094684064	0.1043528	0.1173616	0.1403915	0.1973863
15	0.094134524	0.1043178	0.1183724	0.1416568	0.1953897
16	0.092405163	0.1024797	0.1176013	0.1412600	0.1948619
17	0.090981118	0.1004940	0.1148321	0.1395562	0.2043033
18	0.089875177	0.098631233	0.1119510	0.1374580	0.2003421
19	0.090443790	0.099732995	0.1144853	0.1388062	0.1931063
20	0.089838564	0.099070512	0.1129879	0.1362366	0.1969647
21	0.089684360	0.099597573	0.1121041	0.1365572	0.1876682

Table 4.17. Continued (Mod. C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
22	0.0898[illegible]2	0.099156566	0.1116720	0.1353266	0.1956280
23	0.0892[illegible]3	0.097931020	0.1120457	0.1355183	0.1845801
24	0.090250760	0.098927520	0.1116665	0.1345997	0.1894391
25	0.086477816	0.096268803	0.1097936	0.1330310	0.1883914
26	0.089361295	0.099474996	0.1116120	0.1350340	0.1839367
27	0.087897420	0.096669361	0.1099983	0.1325055	0.1888126
28	0.087610364	0.097102761	0.1115020	0.1350892	0.1910306
29	0.086007208	0.096212253	0.1096980	0.1325047	0.1878328
30	0.086151421	0.095770031	0.1086233	0.1290127	0.1805344
31	0.085716859	0.096077800	0.1094933	0.1336658	0.1866961
32	0.087037697	0.097366460	0.1106475	0.1361363	0.1887586
33	0.086482324	0.095280468	0.1080469	0.1297646	0.1838285
34	0.086857319	0.096330233	0.1084818	0.1334283	0.1921022
35	0.086611502	0.096350551	0.1082830	0.1293105	0.1893613
36	0.085340649	0.093953252	0.1065204	0.1291684	0.1835041
37	0.085253775	0.094642401	0.1082638	0.1305203	0.1826830
38	0.084169395	0.093680479	0.1060074	0.1283366	0.1774915
39	0.084604226	0.093360327	0.1072569	0.1302742	0.1830927

Table 4.18. Continued (Mod. C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
40	0.087177113	0.096347749	0.1103945	0.1337443	0.1834177
41	0.085038245	0.094490454	0.1059541	0.1316031	0.1858505
42	0.084178187	0.093363374	0.1072612	0.1291613	0.1772463
43	0.086647049	0.096142627	0.1099741	0.1314089	0.1859473
44	0.086165667	0.096147276	0.1089791	0.1321802	0.1807189
45	0.086863168	0.097464323	0.1116782	0.1368531	0.1855542
46	0.084294438	0.094070919	0.1069317	0.1279515	0.1944696
47	0.083825737	0.093185492	0.1074729	0.1312952	0.1849413
48	0.084716879	0.093679935	0.1077310	0.1310238	0.1805953
49	0.084885985	0.093539461	0.1057767	0.1298060	0.1897912
50	0.084612489	0.093931831	0.1081517	0.1312491	0.1883270
51	0.085586265	0.095265456	0.1085853	0.1319058	0.1854176
52	0.085252240	0.094202630	0.1075557	0.1286628	0.1816324
53	0.082118019	0.092360727	0.1055925	0.1278450	0.1844971
54	0.084748983	0.093850747	0.1066248	0.1306041	0.1898644
55	0.085000128	0.093799673	0.1080838	0.1314944	0.1822879
56	0.084181368	0.093914039	0.1067785	0.1338640	0.1860174
57	0.082907148	0.091027714	0.1043118	0.1271668	0.1732585

Table 4.19. Continued (Mod. C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
58	0.082885757	0.092641443	0.1053403	0.1266784	0.1769804
59	0.083405033	0.093811668	0.1067430	0.1295628	0.1896901
60	0.083464295	0.093149699	0.1059194	0.1299687	0.1804228
61	0.081543289	0.090840355	0.1036204	0.1264776	0.1685492
62	0.084958017	0.095521927	0.1087770	0.1315241	0.1841160
63	0.082068309	0.092810810	0.1069565	0.1284048	0.1864124
64	0.082610682	0.091001593	0.1046564	0.1266130	0.1744784
65	0.082847416	0.092799872	0.1055391	0.1267786	0.1766667
66	0.081755854	0.090612829	0.1042599	0.1255016	0.1724503
67	0.082927428	0.093032897	0.1063054	0.1294967	0.1764327
68	0.083560646	0.093030259	0.1060542	0.1276196	0.1837067
69	0.083318591	0.093147844	0.1069267	0.1281051	0.1858886
70	0.085421734	0.094470188	0.1080142	0.1290280	0.1830784
71	0.083139651	0.093213074	0.1076766	0.1280835	0.1768681
72	0.083336644	0.092905864	0.1057336	0.1265798	0.1828810
73	0.084840626	0.093278311	0.1057599	0.1302512	0.1799054
74	0.085315138	0.093913630	0.1075264	0.1314406	0.1812564
75	0.084850922	0.094617531	0.1083276	0.1322037	0.1838400

Table 4.20. Continued (Mod. C-VM)

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
76	0.082804367	0.091524869	0.1058075	0.1247443	0.1755374
77	0.082122885	0.091925353	0.1047585	0.1266247	0.1827777
78	0.084189937	0.093079522	0.1064301	0.1291159	0.1803580
79	0.084658489	0.094539940	0.1080402	0.1278412	0.1756680
80	0.083445959	0.093562253	0.1065563	0.1313581	0.1870539

4.2.2 Power Comparison Tables. This section contains two sets of power comparison tables. The first set of tables compares the powers of the tests modified by using the median rank to the powers of the known A-D and CV-M goodness-of-fit tests. The second set of tables compares the powers of the tests modified by using the mean rank to the known A-D and CV-M goodness-of-fit tests. These two sets of power comparisons are presented for each alternative distribution to display the powers of the modified goodness-of-fit tests clearly.

In addition to these power tables, the visual representations for the power improvements are given for each distribution with each plotting position used. These figures show the power improvements as the sample size doubles for the significance level of $\alpha = 0.05$ which is the most commonly used (4).

Table 4.21. Power Comparison for the Anderson-Darling Test with the Uniform Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.2806000	0.2090000	0.1510000	0.0821999	0.0146000
10 ‡	0.2828000	0.2128000	0.1516000	0.0836000	0.0146000
20 †	0.4804000	0.3962000	0.2904000	0.1742000	0.0374000
20 ‡	0.4846000	0.3976000	0.2938000	0.1756000	0.0377999
30 †	0.6394000	0.5648000	0.4526000	0.2906000	0.0828000
30 ‡	0.6436000	0.5676000	0.4564000	0.2932000	0.0842000
40 †	0.7516000	0.6914000	0.6036000	0.4612000	0.07999999
40 ‡	0.7542000	0.6930000	0.6082000	0.4636000	0.1980000
50 †	0.8568000	0.8018000	0.7126000	0.5614000	0.2682000
50 ‡	0.8608000	0.8002000	0.7148000	0.5562000	0.2480000
60 †	0.9146000	0.8826000	0.8278000	0.7072000	0.3774000
60 ‡	0.9154000	0.8764000	0.8194000	0.6864000	0.4166000
70 †	0.9548000	0.9354000	0.9002000	0.8140000	0.5272000
70 ‡	0.9552000	0.9362000	0.9002000	0.8096000	0.5168000
80 †	0.9794000	0.9568000	0.9356000	0.8502000	0.5952000
80 ‡	0.9796000	0.9684000	0.9384000	0.8740000	0.6316000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.22. Power Comparison for the Cramer-von Mises Test with the Uniform Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.2676000	0.2018000	0.1422000	0.0741999	0.1180000
10 ‡	0.2570000	0.1924000	0.1202000	0.0557999	0.0820000
20 †	0.4262000	0.3462000	0.2496000	0.1452000	0.0233999
20 ‡	0.3630000	0.2980000	0.2100000	0.1156000	0.0179999
30 †	0.5560000	0.4688000	0.3714000	0.2342000	0.0575999
30 ‡	0.5180000	0.4344000	0.3376000	0.2164000	0.0575999
40 †	0.6552000	0.5780000	0.4888000	0.3578000	0.1268000
40 ‡	0.6006000	0.5272000	0.4228000	0.2796000	0.0916000
50 †	0.7594000	0.6924000	0.5896000	0.4266000	0.1730000
50 ‡	0.7288000	0.6568000	0.5422000	0.3814000	0.1288000
60 †	0.8388000	0.7820000	0.7056000	0.5468000	0.2878000
60 ‡	0.8080000	0.7470000	0.6604000	0.4984000	0.2272000
70 †	0.8876000	0.8454000	0.7660000	0.6344000	0.3174000
70 ‡	0.8638000	0.8154000	0.7296000	0.5984000	0.2874000
80 †	0.9324000	0.8944000	0.8316000	0.7082000	0.3736000
80 ‡	0.9100000	0.8674000	0.8044000	0.6668000	0.3434000

†Unmodified C-VM Test
‡Modified C-VM Test

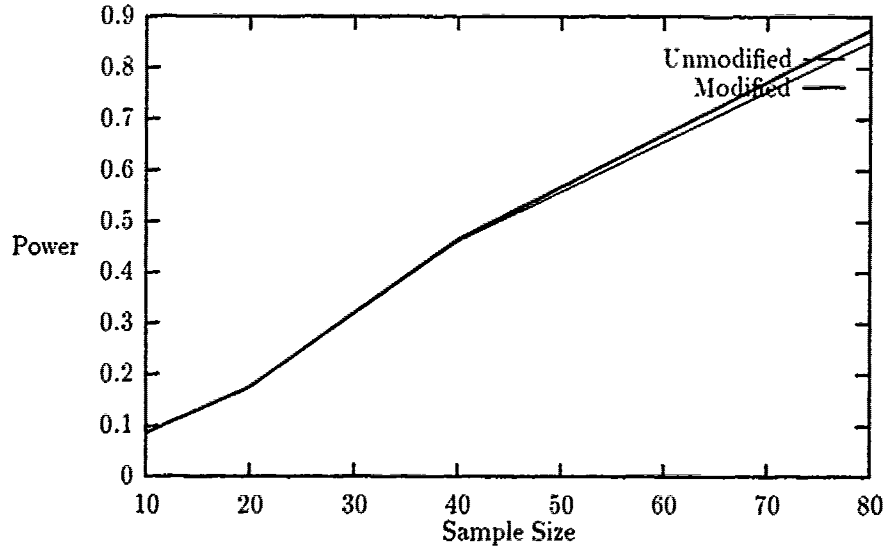

Figure 4.1. Anderson-Darling Power Test for the Uniform Distribution with the Median Rank

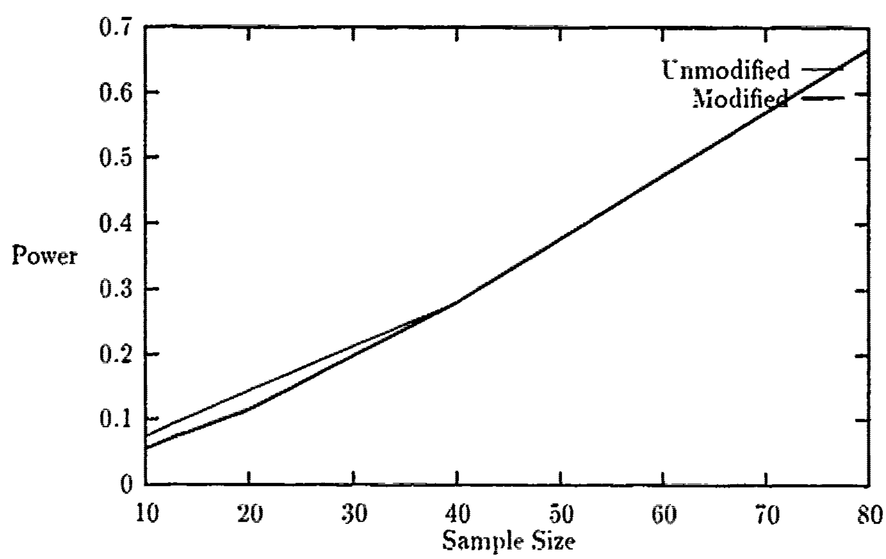

Figure 4.2. Cramer-von Mises Power Test for the Uniform Distribution with the Median Rank

Table 4.23. Power Comparison for the Anderson-Darling Test with the Weibull Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.2020000	0.1518000	0.1082000	0.0572000	0.0114000
10 ‡	0.2020000	0.1520000	0.1094000	0.0577999	0.0116000
20 †	0.2544000	0.1966000	0.1382000	0.0700000	0.0162000
20 ‡	0.2556000	0.1960000	0.1380000	0.0726000	0.0162000
30 †	0.2706000	0.2110000	0.1486000	0.0806000	0.0210000
30 ‡	0.2734000	0.2106000	0.1490000	0.0806000	0.0210000
40 †	0.2976000	0.2358000	0.1774000	0.1072000	0.0288000
40 ‡	0.2970000	0.2364000	0.1784000	0.1082000	0.0289999
50 †	0.3266000	0.2646000	0.1860000	0.1068000	0.0284000
50 ‡	0.3308000	0.2618000	0.1854000	0.1044000	0.0258000
60 †	0.3712000	0.3072000	0.2336000	0.1396000	0.0342000
60 ‡	0.3696000	0.2960000	0.2228000	0.1276000	0.0414000
70 †	0.4006000	0.3398000	0.2624000	0.1648000	0.0511999
70 ‡	0.3998000	0.3390000	0.2606000	0.1612000	0.0496000
80 †	0.4296000	0.3548000	0.2778000	0.1600000	0.0478000
80 ‡	0.4292000	0.3592000	0.2866000	0.1830000	0.0544000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.24. Power Comparison for the Cramer-von Mises Test with the Weibull Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.4380000	0.3768000	0.3062000	0.2192000	0.0939999
10 ‡	0.5484000	0.4824000	0.4030000	0.2978000	0.1266000
20 †	0.6922000	0.6322000	0.5518000	0.4458000	0.2362000
20 ‡	0.7568000	0.7092000	0.6364000	0.5192000	0.3016000
30 †	0.8086000	0.7700000	0.7156000	0.6146000	0.3988000
30 ‡	0.8674000	0.8296000	0.7800000	0.7002000	0.5064000
40 †	0.8996000	0.8694000	0.8272000	0.7596000	0.5714000
40 ‡	0.9288000	0.9034000	0.8646000	0.7862000	0.6176000
50 †	0.9456000	0.9272000	0.8926000	0.8312000	0.6748000
50 ‡	0.9622000	0.9482000	0.9244000	0.8676000	0.7124000
60 †	0.9728000	0.9624000	0.9424000	0.9040000	0.7926000
60 ‡	0.9820000	0.9744000	0.9602000	0.9248000	0.8154000
70 †	0.9846000	0.9786000	0.9650000	0.9364000	0.8398000
70 ‡	0.9894000	0.9854000	0.9758000	0.9538000	0.8756000
80 †	0.9924000	0.9888000	0.9820000	0.9614000	0.8816000
80 ‡	0.9952000	0.9924000	0.9872000	0.9732000	0.9116000

†Unmodified C-VM Test
‡Modified C-VM Test

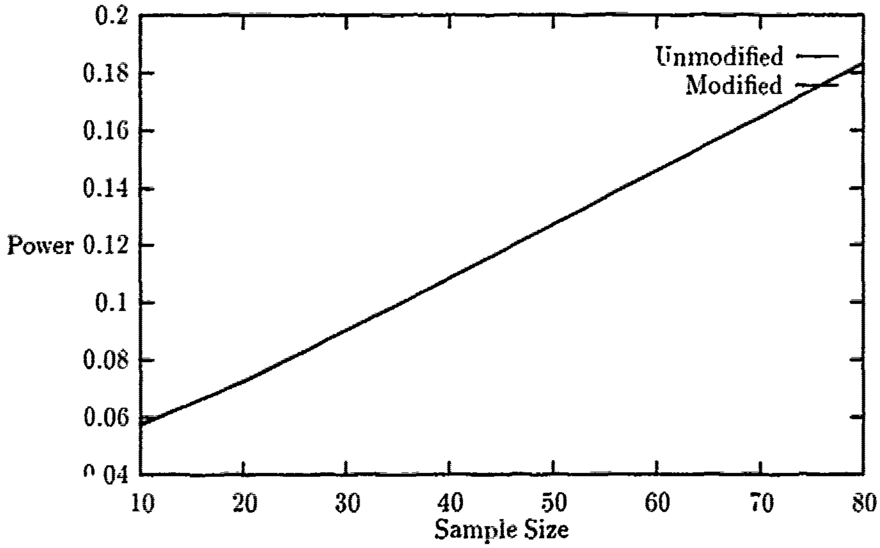

Figure 4.3. Anderson-Darling Power Test for the Weibull Distribution with the Median Rank

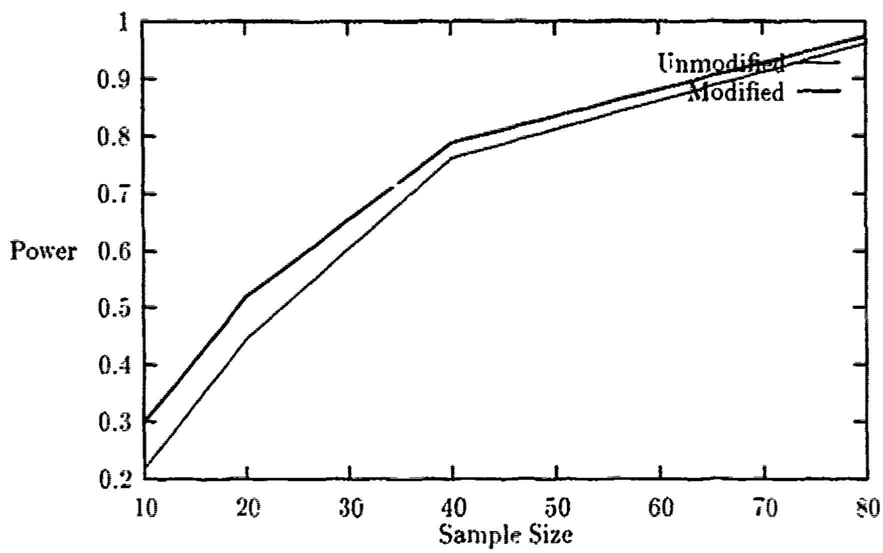

Figure 4.4. Cramer-von Mises Test for the Weibull Distribution with the Median Rank

Table 4.25. Power Comparison for the Anderson-Darling Test with the Exponential Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.6662000	0.6046000	0.5332000	0.4236000	0.2228000
10 ‡	0.6660000	0.6058000	0.5338000	0.4240000	0.2240000
20 †	0.9280000	0.9026000	0.8590000	0.7866000	0.5778000
20 ‡	0.9290000	0.9024000	0.8588000	0.7538000	0.5780000
30 †	0.9868000	0.9822000	0.9682000	0.9366000	0.8128000
30 ‡	0.9868000	0.9822000	0.9684000	0.9366000	0.8120000
40 †	0.9984000	0.9974000	0.9958000	0.9892000	0.9332000
40 ‡	0.9984000	0.9974000	0.9958000	0.9892000	0.9540000
50 †	0.9996000	0.9996000	0.9992000	0.9964000	0.9822000
50 ‡	0.9996000	0.9996000	0.9992000	0.9964000	0.9792000
60 †	0.9996000	0.9996000	0.9996000	0.9992000	0.9956000
60 ‡	0.9996000	0.9996000	0.9996000	0.9992000	0.9964000
70 †	1.000000	1.000000	1.000000	1.000000	0.9990000
70 ‡	1.000000	1.000000	1.000000	1.000000	0.9990000
80 †	1.000000	1.000000	1.000000	1.000000	0.9994000
80 ‡	1.000000	1.000000	1.000000	1.000000	0.9994000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.26. Power Comparison for the Cramer-von Mises Test with the Exponential Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.6402000	0.5746000	0.5044000	0.3898000	0.2012000
10 ‡	0.7354000	0.6808000	0.6026000	0.4912000	0.2494000
20 †	0.8982000	0.8666000	0.8200000	0.7424000	0.4960000
20 ‡	0.9306000	0.9074000	0.8690000	0.7958000	0.5900000
30 †	0.9762000	0.9644000	0.9466000	0.9010000	0.7490000
30 ‡	0.9874000	0.9800000	0.9666000	0.9392000	0.8300000
40 †	0.9958000	0.9928000	0.9880000	0.9754000	0.9112000
40 ‡	0.9976000	0.9962000	0.9926000	0.9814000	0.9312000
50 †	0.9990000	0.9990000	0.9968000	0.9910000	0.9640000
50 ‡	0.9994000	0.9992000	0.9982000	0.9948000	0.9716000
60 †	0.9992000	0.9992000	0.9988000	0.9976000	0.9916000
60 ‡	0.9996000	0.9994000	0.9992000	0.9986000	0.9930000
70 †	1.000000	1.000000	1.000000	1.000000	0.9958000
70 ‡	1.000000	1.000000	1.000000	1.000000	0.9966000
80 †	1.000000	1.000000	1.000000	0.9996000	0.9982000
80 ‡	1.000000	1.000000	1.000000	0.9998000	0.9988000

†Unmodified C-VM Test
‡Modified C-VM Test

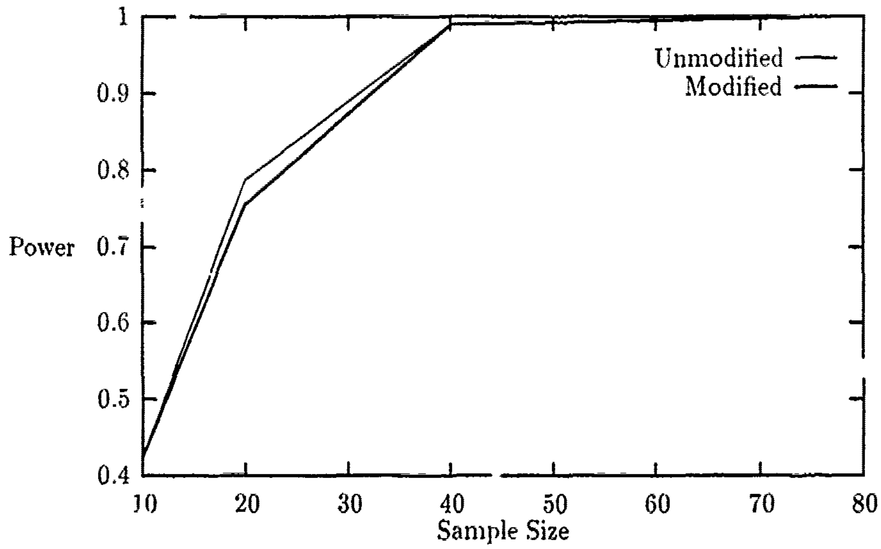

Figure 4.5. Anderson-Darling Power Test for the Exponential Distribution with the Median Rank

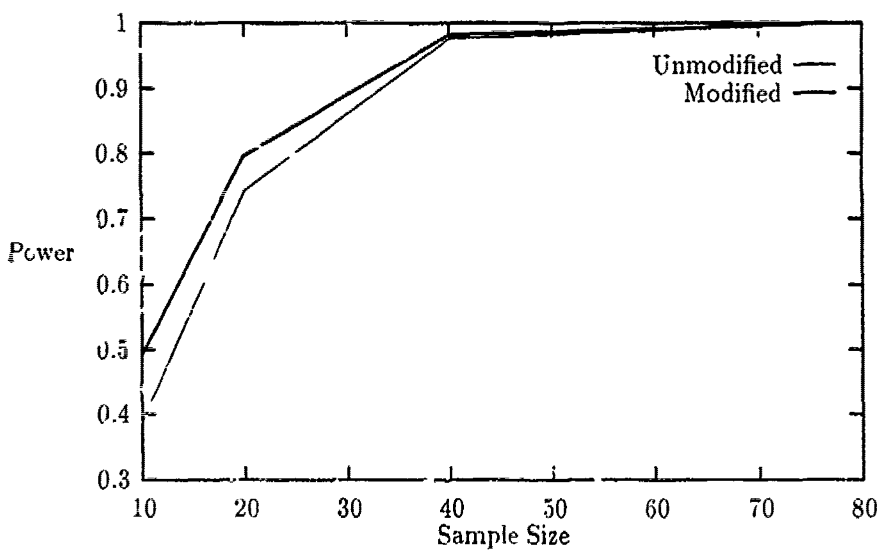

Figure 4.6. Cramer von Mises Power Test for the Exponential Distribution with the Median Rank

Table 4.27. Power Comparison for the Anderson-Darling Test with the Lognormal Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.4562000	0.3890000	0.3210000	0.2356000	0.1046000
10 ‡	0.4550000	0.3908000	0.3220000	0.2354000	0.1048000
20 †	0.7246000	0.6708000	0.5950000	0.4808000	0.2860000
20 ‡	0.7252000	0.6694000	0.5948000	0.4420000	0.2850000
30 †	0.8392000	0.8050000	0.7536000	0.6518000	0.4524000
30 ‡	0.8402000	0.8050000	0.7530000	0.6512000	0.4514000
40 †	0.9266000	0.9016000	0.8700000	0.8056000	0.1656000
40 ‡	0.9268000	0.9016000	0.8706000	0.8044000	0.6348000
50 †	0.9590000	0.9466000	0.9238000	0.8712000	0.7346000
50 ‡	0.9604000	0.9458000	0.9236000	0.8682000	0.7234000
60 †	0.9826000	0.975800	0.9462000	0.9352000	0.8244000
60 ‡	0.9818000	0.9742000	0.9612000	0.9304000	0.8402000
70 †	0.9912000	0.9878000	0.9810000	0.9660000	0.9034000
70 ‡	0.9910000	0.9878000	0.9804000	0.9650000	0.8998000
80 †	0.9976000	0.9950000	0.9912000	0.9802000	0.9290000
80 ‡	0.9976000	0.9952000	0.9920000	0.9822000	0.9380000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.28. Power Comparison for the Cramer-von Mises Test with the Lognormal Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.4450000	0.3772000	0.3080000	0.2112000	0.0726000
10 ‡	0.5966000	0.5294000	0.4468000	0.3300000	0.1488000
20 †	0.6922000	0.6322000	0.5518000	0.4458000	0.2362000
20 ‡	0.7568000	0.7092000	0.6364000	0.5192000	0.3016000
30 †	0.8220000	0.7780000	0.7196000	0.6242000	0.3896000
30 ‡	0.8516000	0.8186000	0.7630000	0.6710000	0.4468000
40 †	0.9060000	0.87[illegible]	0.8278000	0.7504000	0.5286000
40 ‡	0.9194000	0.8970000	0.8562000	0.7780000	0.5710000
50 †	0.9490000	0.9296000	0.8978000	0.8422000	0.6568000
50 ‡	0.9530000	0.9392000	0.9154000	0.8564000	0.6822000
60 †	0.9738000	0.9630000	0.9420000	0.9064000	0.7590000
60 ‡	0.9770000	0.9682000	0.9514000	0.9146000	0.7794000
70 †	0.9862000	0.9800000	0.9676000	0.9438000	0.8314000
70 ‡	0.9866000	0.9838000	0.9716000	0.9468000	0.8434000
80 †	0.9930000	0.9894000	0.9836000	0.9664000	0.8850000
80 ‡	0.9936000	0.9906000	0.9846000	0.9684000	0.8948000

†Unmodified C-VM Test
‡Modified C-VM Test

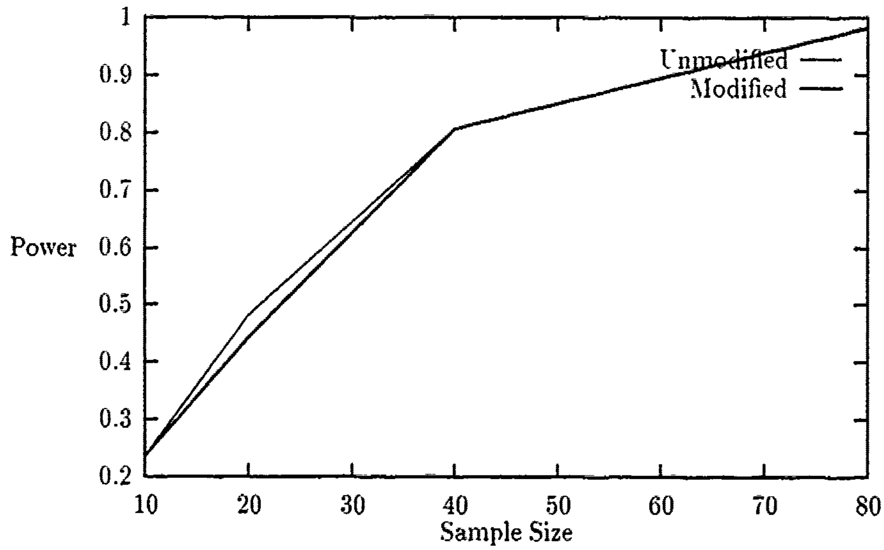

Figure 4.7. Anderson-Darling Power Test for the Lognormal Distribution with the Median Rank

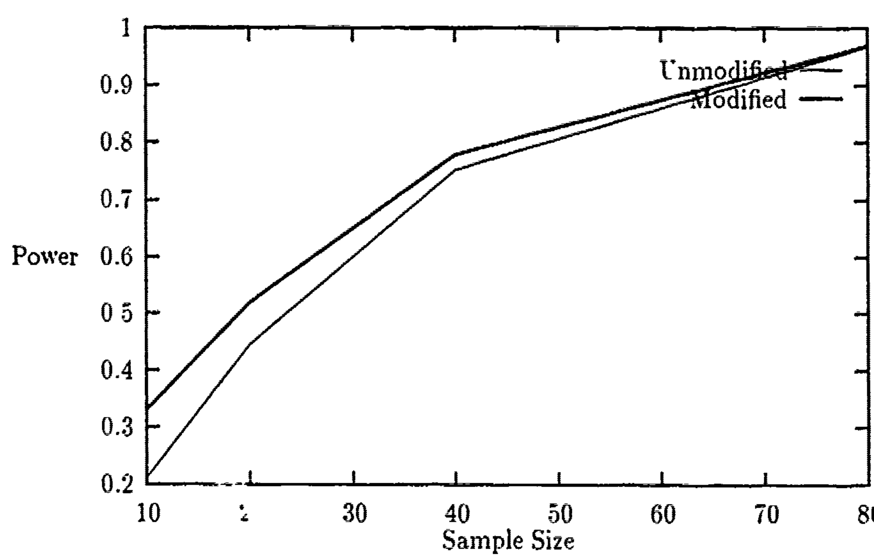

Figure 4.8. Cramer-von Mises Power Test for the Lognormal Distribution with the Median Rank

Table 4.29. Power Comparison for the Anderson-Darling Test with the Double Exponential Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.7352000	0.6798000	0.6130000	0.5002000	0.2878000
10 ‡	0.7358000	0.6810000	0.6132000	0.5004000	0.2888000
20 †	0.9588000	0.9440000	0.9144000	0.8618000	0.6896000
20 ‡	0.9592000	0.9440000	0.9150000	0.8354000	0.6894000
30 †	0.9950000	0.9918000	0.9856000	0.9666000	0.8968000
30 ‡	0.9950000	0.9918000	0.9858000	0.9664000	0.8960000
40 †	0.9996000	0.9996000	0.9992000	0.9964000	0.7180000
40 ‡	0.9996000	0.9996000	0.9992000	0.9964000	0.9774000
50 †	1.000000	1.000000	1.000000	0.9992000	0.9950000
50 ‡	1.000000	1.000000	1.000000	0.9992000	0.9938000
60 †	1.000000	1.000000	1.000000	1.000000	0.9992000
60 ‡	1.000000	1.000000	1.000000	1.000000	0.9996000
70 †	1.000000	1.000000	1.000000	1.000000	1.000000
70 ‡	1.000000	1.000000	1.000000	1.000000	1.000000
80 †	1.000000	1.000000	1.000000	1.000000	1.000000
80 ‡	1.000000	1.000000	1.000000	1.000000	1.000000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.30. Power Comparison for the Cramer-von Mises Test with the Double Exponential Distribution Using the Median Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.7104000	0.6526000	0.5848000	0.4662000	0.2624000
10 ‡	0.7926000	0.7512000	0.6792000	0.5678000	0.3138000
20 †	0.9422000	0.9210000	0.8834000	0.8258000	0.6158000
20 ‡	0.9606000	0.9456000	0.92[illegible]000	0.8670000	0.6880000
30 †	0.9890000	0.9834000	0.9730000	0.9464000	0.8556000
30 ‡	0.0.9944000	0.9904000	0.9840000	0.9690000	0.9020000
40 †	0.9984000	0.9976000	0.9952000	0.9894000	0.9598000
40 ‡	0.9996000	0.9990000	0.9976000	0.9912000	0.9688000
50 †	1.000000	1.000000	0.9986000	0.9966000	0.9854000
50 ‡	1.000000	1.000000	1.000000	0.9984000	0.9888000
60 †	1.000000	1.000000	1.000000	0.9996000	0.9978000
60 ‡	1.000000	1.000000	1.000000	0.9998000	0.9986000
70 †	1.000000	1.000000	1.000000	1.000000	0.9992000
70 ‡	1.000000	1.000000	1.000000	1.000000	0.9996000
80 †	1.000000	1.000000	1.000000	1.000000	0.9998000
80 ‡	1.000000	1.000000	1.000000	1.000000	1.000000

†Unmodified C-VM Test
‡Modified C-VM Test

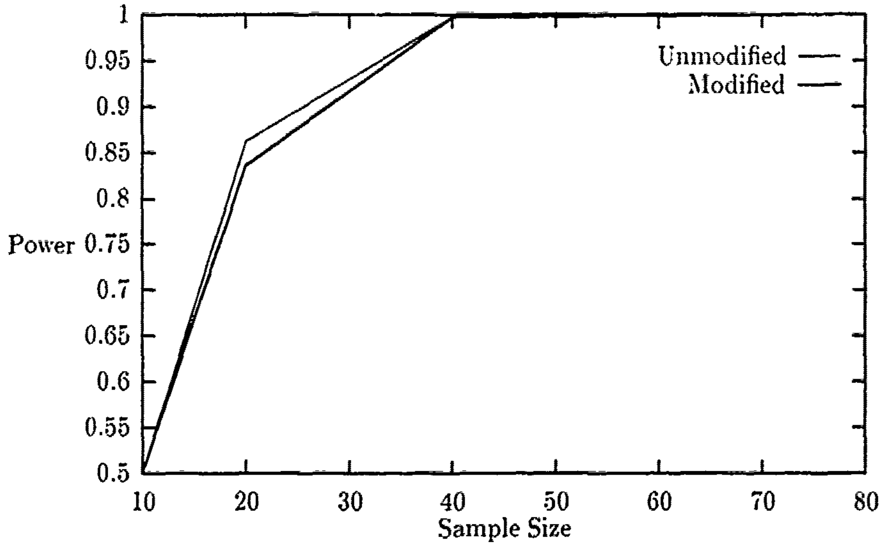

Figure 4.9. Anderson-Darling Power Test for the Double Exponential Distribution with the Median Rank

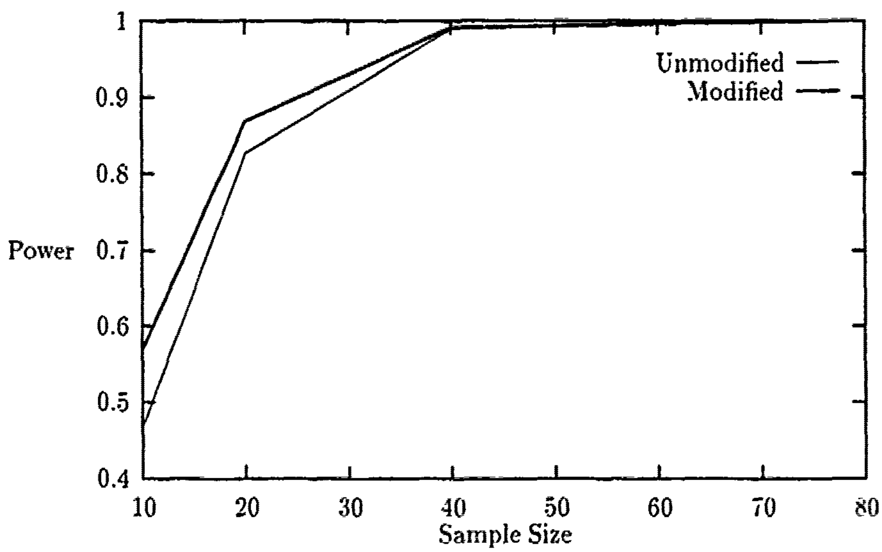

Figure 4.10. Cramer-von Mises Power Test for the Double Exponential Distribution with the Median Rank

Table 4.31. Power Comparison for the Anderson-Darling Test with the Uniform Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.2806000	0.2090000	0.1510000	0.0821999	0.0146000
10 ‡	0.2840000	0.2160000	0.1566000	0.0858000	0.0152000
20 †	0.4804000	0.3962000	0.2904000	0.1742000	0.0374000
20 ‡	0.5103000	0.4012000	0.3124000	0.1891000	0.0472100
30 †	0.6394000	0.5648000	0.4526000	0.2906000	0.0828000
30 ‡	0.6488000	0.5770000	0.4660000	0.3024000	0.0869999
40 †	0.7516000	0.6914000	0.6036000	0.4612000	0.0799999
40 ‡	0.7610000	0.6962000	0.5996000	0.4178000	0.1878000
50 †	0.8568000	0.8018000	0.7126000	0.5614000	0.2682000
50 ‡	0.8686000	0.8296000	0.7520000	0.6136000	0.3008000
60 †	0.9146000	0.8826000	0.8278000	0.7072000	0.3774000
60 ‡	0.9100000	0.8716000	0.8078000	0.6766000	0.3626000
70 †	0.9548000	0.9354000	0.9002000	0.8140000	0.5272000
70 ‡	0.9552000	0.9346000	0.8936000	0.7948000	0.4902000
80 †	0.9794000	0.9658000	0.9356000	0.8502000	0.5952000
80 ‡	0.9796000	0.9672000	0.9356000	0.8626000	0.5714000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.32. Power Comparison for the Cramer-von Mises Test with the Uniform Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.2676000	0.2018000	0.1422000	0.0741000	0.0118000
10 ‡	0.3312000	0.2688000	0.1904000	0.1056000	0.0223999
20 †	0.4262000	0.3462000	0.2496000	0.1452000	0.0233999
20 ‡	0.5166000	0.4310000	0.3304000	0.2078000	0.0524000
30 †	0.5560000	0.4688000	0.3714000	0.2342000	0.0575999
30 ‡	0.6498000	0.5706000	0.4640000	0.3172000	0.0913999
40 †	0.6552000	0.5780000	0.4888000	0.3578000	0.1268000
40 ‡	0.7334000	0.6692000	0.5752000	0.4420000	0.1850000
50 †	0.7594000	0.6924000	0.5896000	0.4266000	0.1730000
50 ‡	0.8312000	0.7588000	0.6726000	0.5138000	0.2420000
60 †	0.8388000	0.7820000	0.7056000	0.5468000	0.2878000
60 ‡	0.8838000	0.8394000	0.7708000	0.6260000	0.3556000
70 †	0.8876000	0.8454000	0.7660000	0.6344000	0.3174000
70 ‡	0.9236000	0.8880000	0.8224000	0.6984000	0.3814000
80 †	0.9324000	0.8944000	0.8316000	0.7082000	0.3736000
80 ‡	0.9528000	0.9206000	0.8710000	0.7758000	0.4688000

†Unmodified C-VM Test
‡Modified C-VM Test

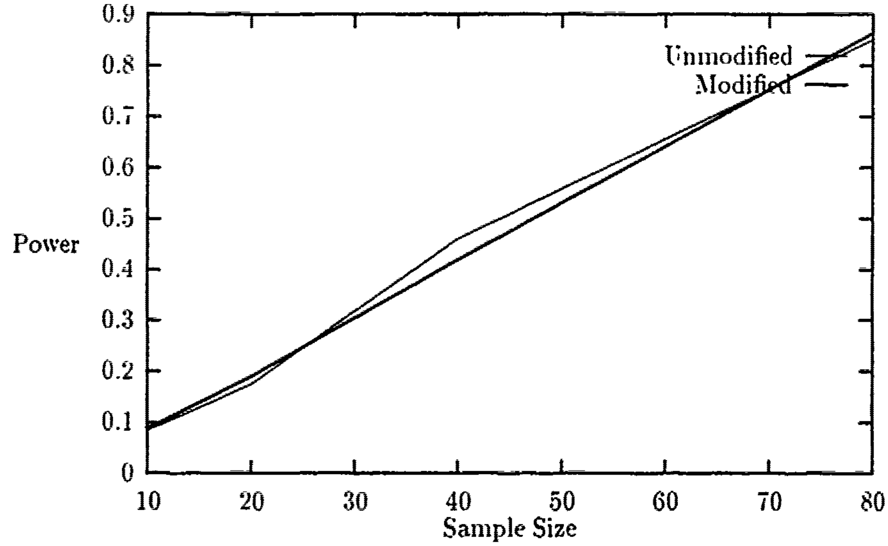

Figure 4.11. Anderson-Darling Power Test for the Uniform Distribution with the Mean Rank

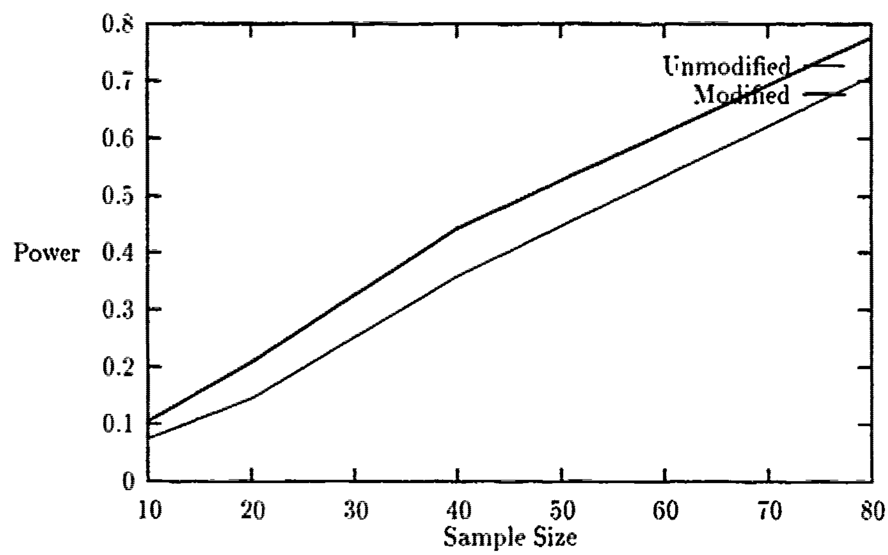

Figure 4.12. Cramer-von Mises Power Test for the Uniform Distribution with the Mean Rank

Table 4.33. Power Comparison for the Anderson-Darling Test with the Weibull Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.2020000	0.1518000	0.1082000	0.0572000	0.0114000
10 ‡	0.2020000	0.1538000	0.1106000	0.0579999	0.0124000
20 †	0.2544000	0.1966000	0.1382000	0.0700000	0.0162000
20 ‡	0.2610000	0.1988000	0.1400000	0.0761000	0.0167200
30 †	0.2706000	0.2110000	0.1486000	0.0806000	0.0210000
30 ‡	0.2748000	0.2132000	0.1520000	0.0816000	0.0208000
40 †	0.2976000	0.2358000	0.1774000	0.1074200	0.0297000
40 ‡	0.2996000	0.2352000	0.1712000	0.0902000	0.0254000
50 †	0.3266000	0.2646000	0.1860000	0.1068000	0.0284000
50 ‡	0.3452000	0.2876000	0.2126000	0.1256000	0.0335999
60 †	0.3712000	0.3072000	0.2336000	0.1396000	0.0342000
60 ‡	0.3522000	0.2846000	0.2064000	0.1194000	0.0304000
70 †	0.4006000	0.3398000	0.2624000	0.1648000	0.0511999
70 ‡	0.3964000	0.3310000	0.2490000	0.1490000	0.0440000
80 †	0.4296000	0.3548000	0.2778000	0.1600000	0.0478000
80 ‡	0.4246000	0.3536000	0.2722000	0.1686000	0.0397999

†Unmodified A-D Test
‡Modified A-D Test

Table 4.34. Power Comparison for the Cramer-von Mises Test with the Weibull Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.1986000	0.1558000	0.1092000	0.0562000	0.0112000
10 ‡	0.2154000	0.1662000	0.1130000	0.0553999	0.0124000
20 †	0.2482000	0.1922000	0.1290000	0.0692000	0.0124000
20 ‡	0.2628000	0.2016000	0.1370000	0.0777999	0.0140000
30 †	0.2630000	0.2028000	0.1408000	0.0766000	0.0189999
30 ‡	0.2740000	0.2204000	0.1506000	0.0839999	0.0206000
40 †	0.2828000	0.2154000	0.1638000	0.1022000	0.0250000
40 ‡	0.2942000	0.2380000	0.1768000	0.1060000	0.0280000
50 †	0.3140000	0.2472000	0.1752000	0.1004000	0.0255999
50 ‡	0.3378000	0.2622000	0.1856000	0.1076000	0.0286000
60 †	0.3474000	0.2822000	0.2108000	0.1180000	0.0392000
60 ‡	0.3646000	0.2982000	0.2286000	0.1238000	0.0388000
70 †	0.3530000	0.2906000	0.2088000	0.1302000	0.0399999
70 ‡	0.3728000	0.3002000	0.2236000	0.1352000	0.0392000
80 †	0.3978000	0.3208000	0.2458000	0.1428000	0.0357999
80 ‡	0.3976000	0.3234000	0.2482000	0.1516000	0.0399999

†Unmodified C-VM Test
‡Modified C-VM Test

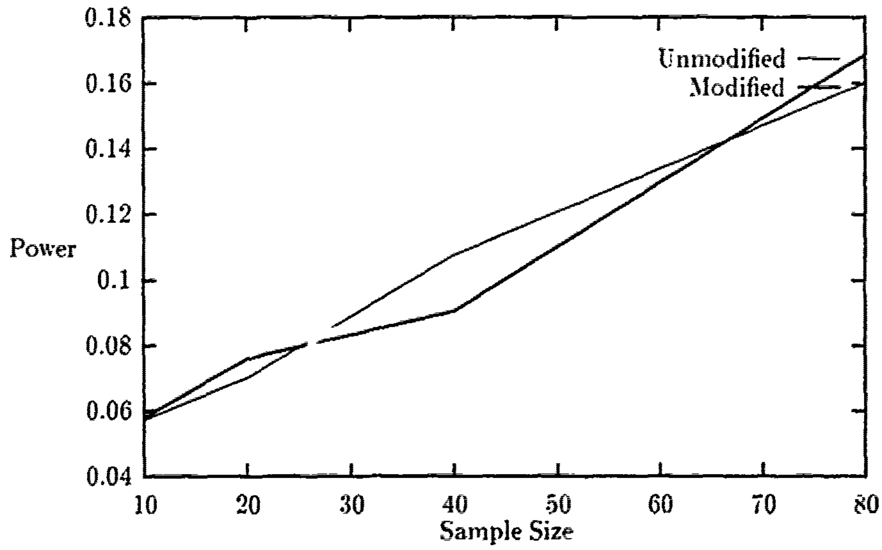

Figure 4.13. Anderson-Darling Power Test for the Weibull Distribution with the Mean Rank

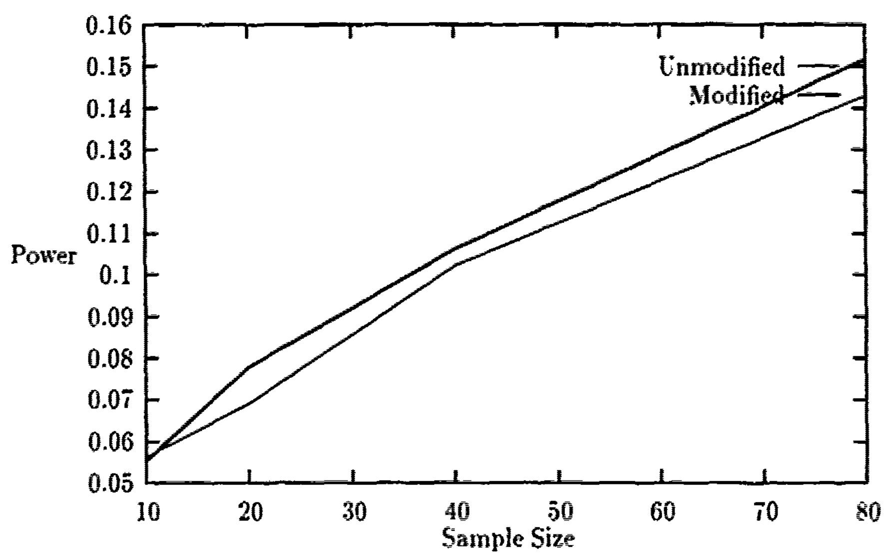

Figure 4.14. Cramer-von Mises Power Test for the Weibull Distribution with the Mean Rank

Table 4.35. Power Comparison for the Anderson-Darling Test with the Exponential Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.6662000	0.6046000	0.5332000	0.4236000	0.2228000
10 ‡	0.6656000	0.6062000	0.5358000	0.4260000	0.2234000
20 †	0.9280000	0.9026000	0.8590000	0.7866000	0.5778000
20 ‡	0.930000	0.9028000	0.8610000	0.7930000	0.5800000
30 †	0.9868000	0.9822000	0.9682000	0.9366000	0.8128000
30 ‡	0.9868000	0.9822000	0.9694000	0.9368000	0.8120000
40 †	0.9984000	0.9974000	0.9958000	0.9892000	0.9332000
40 ‡	0.9984000	0.9974000	0.9948000	0.9842000	0.9500000
50 †	0.9996000	0.9996000	0.9992000	0.9964000	0.9822000
50 ‡	0.9996000	0.9996000	0.9994000	0.9976000	0.9854000
60 †	0.9996000	0.9996000	0.9996000	0.9[illegible]2000	0.9956000
60 ‡	0.9996000	0.9996000	0.9996000	0.9990000	0.9950000
70 †	1.000000	1.000000	1.000000	1.000000	0.9990000
70 ‡	1.000000	1.000000	1.000000	1.000000	0.9990000
80 †	1.000000	1.000000	1.000000	1.000000	0.9994000
80 ‡	1.[illegible]0000	1.000000	1.000000	1.000000	0.9994000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.36. Power Comparison for the Cramer-von Mises Test with the Exponential Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.6402000	0.5746000	0.5044000	0.3898000	0.2012000
10 ‡	0.6352000	0.5734000	0.4938000	0.3766000	0.1952000
20 †	0.8982000	0.8666000	0.8200000	0.7424000	0.4960000
20 ‡	0.8940000	0.8624000	0.8134000	0.7384000	0.5072000
30 †	0.9762000	0.9644000	0.9466000	0.9010000	0.7490000
30 ‡	0.9760000	0.9652000	0.9446000	0.9004000	0.7406000
40 †	0.9958000	0.9928000	0.9880000	0.9754000	0.9112000
40 ‡	0.9960000	0.9932000	0.9872000	0.9740000	0.9084000
50 †	0.9990000	0.9990000	0.9968000	0.9910000	0.9640000
50 ‡	0.9990000	0.9988000	0.9970000	0.9906000	0.9640000
60 †	0.9992000	0.9992000	0.9988000	0.9976000	0.9916000
60 ‡	0.9992000	0.9992000	0.9988000	0.9978000	0.9908000
70 †	1.000000	1.000000	1.000000	1.000000	0.9958000
70 ‡	1.000000	1.000000	1.000000	1.000000	0.9958000
80 †	1.000000	1.000000	1.000000	0.9996000	0.9982000
80 ‡	1.000000	1.000000	1.000000	0.9996000	0.9984000

†Unmodified C-VM Test
‡Modified C-VM Test

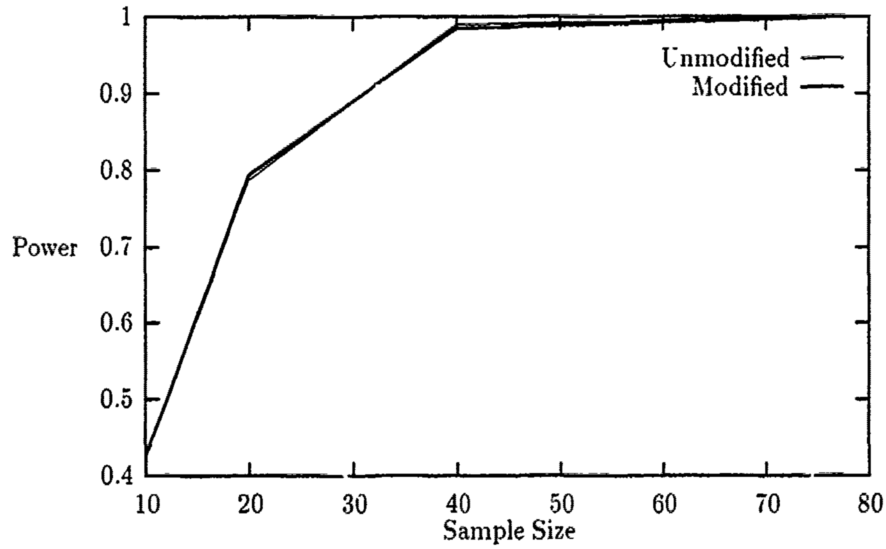

Figure 4.15. Anderson-Darling Power Test for the Exponential Distribution with the Mean Rank

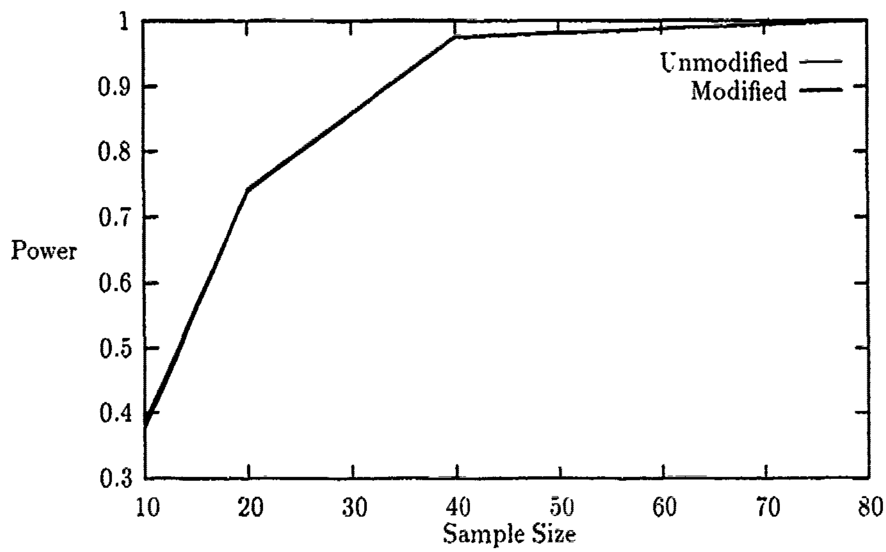

Figure 4.16. Cramer-von Mises Test for the Exponential Distribution with the Mean Rank

Table 4.37. Power Comparison for the Anderson-Darling Test with the Double Exponential Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.7352000	0.6798000	0.6130000	0.5002000	0.2878000
10 ‡	0.7352000	0.6814000	0.6138000	0.5012000	0.2884000
20 †	0.9588000	0.9440000	0.9144000	0.8618000	0.6896000
20 ‡	0.9588000	0.9442000	0.9146000	0.8700000	0.6898000
30 †	0.9950000	0.9918000	0.9856000	0.9666000	0.8968000
30 ‡	0.9950000	0.9918000	0.9860000	0.9664000	0.8956000
40 †	0.9996000	0.9996000	0.9992000	0.9964000	0.7180000
40 ‡	0.9996000	0.9996000	0.9992000	0.9946000	0.9748000
50 †	1.000000	1.000000	1.000000	0.9992000	0.9950000
50 ‡	1.000000	1.000000	1.000000	0.9996000	0.9962000
60 †	1.000000	1.000000	1.000000	1.000000	0.9992000
60 ‡	1.000000	1.000000	1.000000	1.000000	0.9996000
70 †	1.000000	1.000000	1.000000	1.000000	1.000000
70 ‡	1.000000	1.000000	1.000000	1.000000	0.9998000
80 †	1.000000	1.000000	1.000000	1.000000	1.000000
80 ‡	1.000000	1.000000	1.000000	1.000000	1.000000

†Unmodified A-D Test
‡Modified A-D Test

Table 4.38. Power Comparison for the Cramer-von Mises Test with the Double Exponential Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.7104000	0.6526000	0.5848000	0.4662000	0.2624000
10 ‡	0.7064000	0.6478000	0.5692000	0.4502000	0.2582000
20 †	0.9422000	0.9210000	0.8834000	0.8258000	0.6158000
20 ‡	0.9400000	0.9174000	0.8802000	0.8174000	0.6158000
30 †	0.9890000	0.9834000	0.9730000	0.9464000	0.8556000
30 ‡	0.9892000	0.9838000	0.9718000	0.9444000	0.8486000
40 †	0.9984000	0.9976000	0.9952000	0.9894000	0.9598000
40 ‡	0.9980000	0.9970000	0.9946000	0.9886000	0.9582000
50 †	1.000000	1.000000	0.9986000	0.9966000	0.9854000
50 ‡	1.000000	0.9996000	0.9988000	0.9966000	0.9860000
60 †	1.000000	1.000000	1.000000	0.9996000	0.9978000
60 ‡	1.000000	1.000000	1.000000	0.9996000	0.9968000
70 †	1.000000	1.000000	1.000000	1.000000	0.9992000
70 ‡	1.000000	1.000000	1.000000	1.000000	0.9992000
80 †	1.000000	1.000000	1.000000	1.000000	0.9998000
80 ‡	1.000000	1.000000	1.000000	1.000000	1.000000

†Unmodified C-VM Test
‡Modified C-VM Test

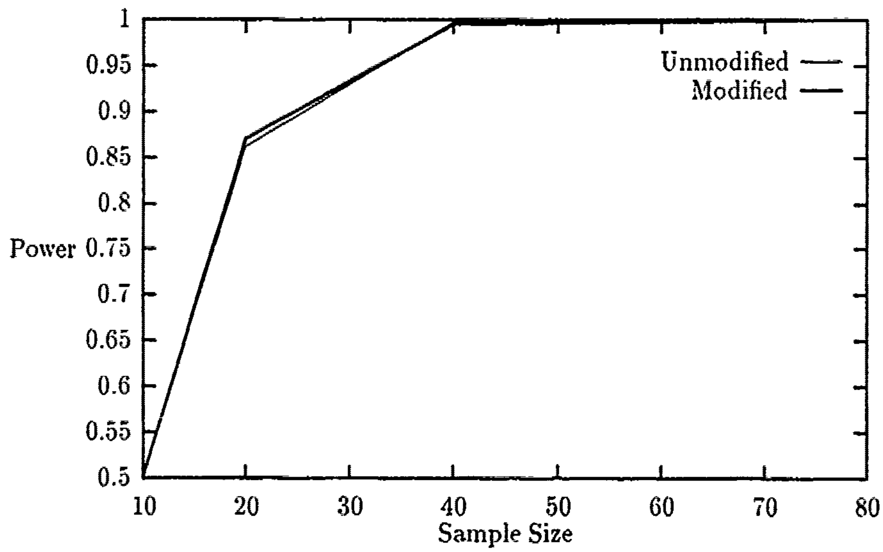

Figure 4.17. Anderson-Darling Power Test for the Double Exponential Distribution with the Mean Rank

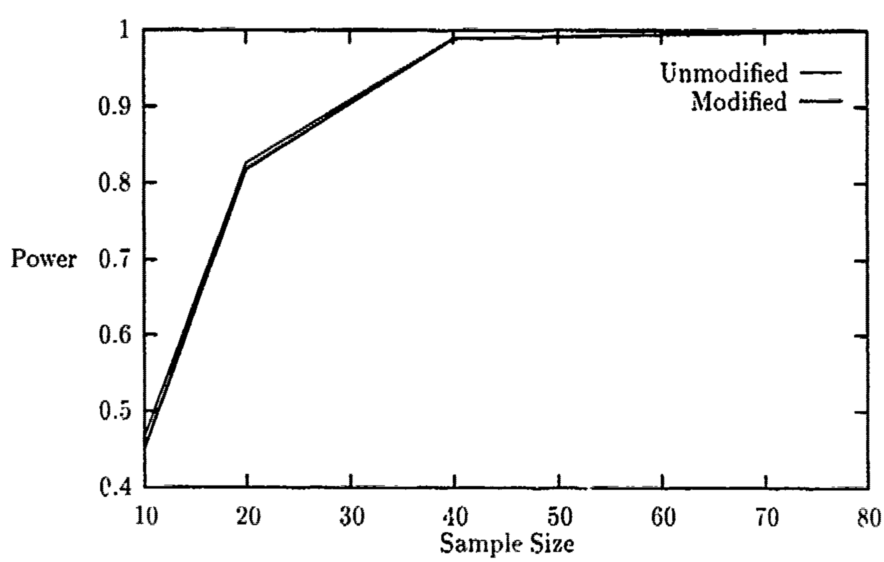

Figure 4.18. Cramer-von Mises Power Test for the Double Exponential Distribution with the Mean Rank

Table 4.39. Power Comparison for the Anderson-Darling Test with the Lognormal Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.4562000	0.3890000	0.3218000	0.2356000	0.1046000
10 ‡	0.4534000	0.3896000	0.3220000	0.2354000	0.1048000
20 †	0.7246000	0.6708000	0.5950000	0.4808000	0.2860000
20 ‡	0.7300000	0.6711000	0.5990000	0.4812000	0.2861000
30 †	0.8392000	0.8050000	0.7536000	0.6518000	0.4524000
30 ‡	0.8394000	0.8040000	0.7534000	0.6518000	0.4482000
40 †	0.9266000	0.9016000	0.8700000	0.8056000	0.6156000
40 ‡	0.9268000	0.9006000	0.8636000	0.7760000	0.6192000
50 †	0.9590000	0.9466000	0.9238000	0.8712000	0.7346000
50 ‡	0.9616000	0.9516000	0.9318000	0.8864000	0.7490000
60 †	0.9826000	0.9758000	0.9642000	0.9352000	0.8244000
60 ‡	0.9794000	0.9726000	0.9582000	0.9254000	0.8128000
70 †	0.9912000	0.9878000	0.9810000	0.9660000	0.9034000
70 ‡	0.9906000	0.9870000	0.9792000	0.9602000	0.8910000
80 †	0.9976000	0.9950000	0.9912000	0.9802000	0.9290000
80 ‡	0.9974000	0.9946000	0.9902000	0.9806000	0.9222000

†Unmodified A-D Test
‡Modified Test

Table 4.40. Power Comparison for the Cramer-von Mises Test with the Lognormal Distribution Using the Mean Rank

	Level of Significance (α= Type I Error)				
n	0.20	0.15	0.10	0.05	0.01
10 †	0.4380000	0.3768000	0.3062000	0.2192000	0.0939999
10 ‡	0.4282000	0.3642000	0.2912000	0.2004000	0.0851999
20 †	0.6922000	0.6322000	0.5518000	0.4458000	0.2362000
20 ‡	0.6790000	0.6124000	0.5408000	0.4288000	0.2294000
30 †	0.8086000	0.7700000	0.7156000	0.6146000	0.3988000
30 ‡	0.8010000	0.7586000	0.7036000	0.6050000	0.3814000
40 †	0.8996000	0.8694000	0.8272000	0.7596000	0.5714000
40 ‡	0.8936000	0.8666000	0.8206000	0.7430000	0.5574000
50 †	0.9456000	0.9272000	0.8926000	0.8312000	0.6748000
50 ‡	0.9438000	0.9202000	0.8872000	0.8220000	0.6678000
60 †	0.9728000	0.9624000	0.9424000	0.9040000	0.7926000
60 ‡	0.9718000	0.9596000	0.9408000	0.8992000	0.7778000
70 †	0.9846000	0.9786000	0.9650000	0.9364000	0.8398000
70 ‡	0.9840000	0.9758000	0.9632000	0.9334000	0.8294000
80 †	0.9924000	0.9888000	0.9820000	0.9614000	0.8816000
80 ‡	0.9914000	0.9884000	0.9798000	0.9602000	0.8824000

†Unmodified C-VM Test
‡Modified C-VM Test

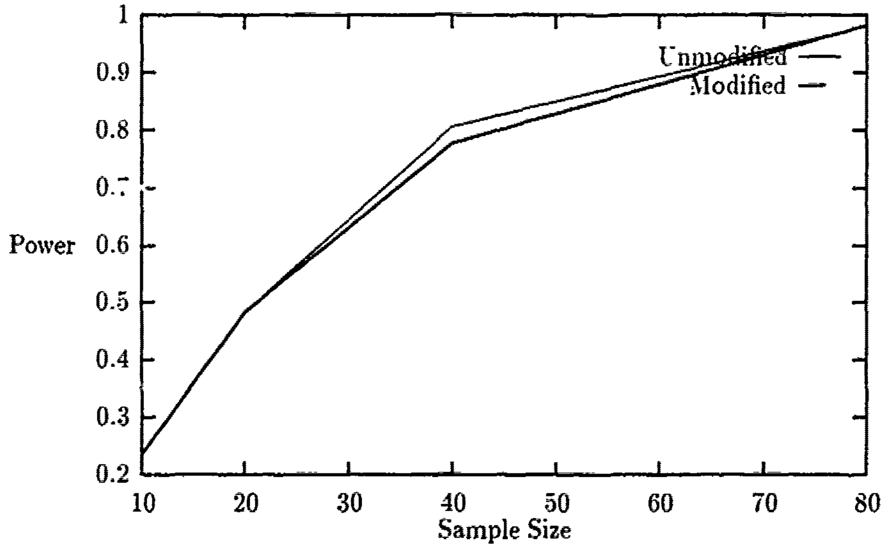

Figure 4.19. Anderson-Darling Power Test for the Lognormal Distribution with the Mean Rank

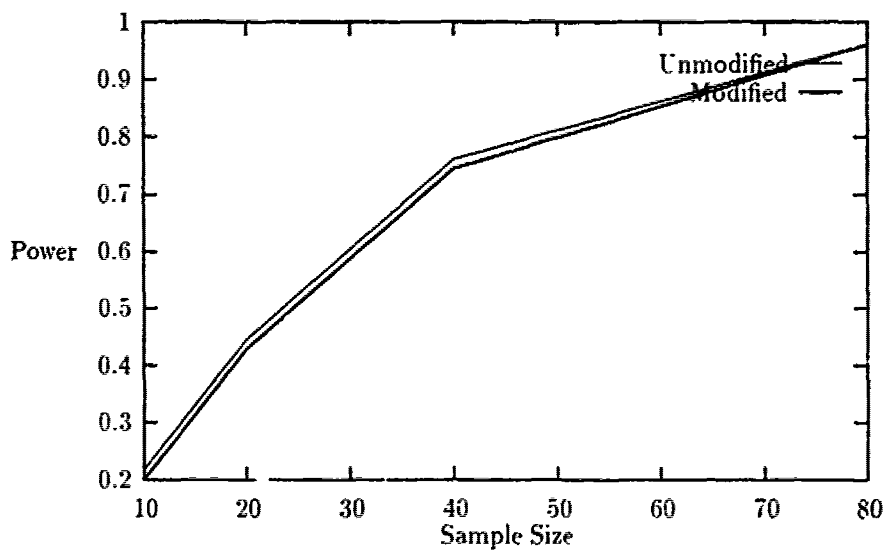

Figure 4.20. Cramer-von Mises Power Test for the Lognormal Distribution with the Median Rank

4.3 Conclusion

This chapter is mainly a collection of tables. The critical values are presented for the known and the modified tests for the sample sizes $n = 4$ through $n = 80$ at the significance levels of $\alpha = .20, .15, .10, .05, .01$. The power comparison tables contain two sets of tables. These compare the powers of the modified tests to the known A-D and CV-M tests. The results of the power comparisons will be given in the next chapter.

V. *Results*

5.1 *Introduction*

This chapter discusses the results of the power comparisons presented in the previous chapter and the computer programs used. Specifically, the discussion covers the topics of the computer programs, the test of the random number generator, the validation and verification, and the analysis of the power comparisons.

5.2 *Discussion*

5.2.1 Computer Programs. There are two main programs used in the study. One of them is for calculating the critical values, and the other is for performing the power comparisons. Both programs are written in FORTRAN 77 and are run in AFITNET Mainframe. In addition to these programs, some subroutines are taken from the International Mathematical and Statistics Library(IMSL). Both of these programs are presented in the Appendices. The detailed comments are given to help the readers understand the logic of the programs clearly. The flow diagrams for the computer programs are given in Chapter 3.

5.2.2 Test of the Random Number Generator. To generate the critical value tables precisely, it is essential to use a reliable random number generator. Forty numbers were generated by using the subroutine GGNO from the International Mathematical and Statistics Library (IMSL). GGNO generates an array of ordered N(0,1) random deviates. The method of normal probability plotting was used to investigate the goodness-of-fit of these deviates generated to the normal distribution. After the deviates were plotted, a line was drawn "by eye" through these points (6:28). Two points on the plot corresponding to around the 10^{th} percentile and another around the 90^{th} percentile were located and connected to each other. This line showed a very good fit.

5.2.3 Validation and Verification. The validation of the computer programs is performed by comparing the critical values to the other published critical values. The critical

values for the known A-D and C-VM tests were compared to the ones obtained by Ream (17). The values were the same for the unmodified goodness-of-fit tests.

The purpose of the verification is to ensure that the concepts and the equations derived in the study are accurately reflected by the computer programs. To achieve this, every line of the computer programs was checked many times by the author.

5.2.4 The Analysis of Power Comparisons. Using the median rank plotting position improved the power of the C-VM goodness-of-fit test approximately 10% for the lognormal, the exponential, the Weibull, and the double exponential.

Using the mean rank plotting position also improved the power of the C-VM goodness-of-fit test only for the uniform and the Weibull distributions; while it reduced the powers for the other three distributions. The improvement for the uniform distribution was 9%; while the improvement was 2% for the Weibull distribution.

Using the median rank plotting position improved the power of the A-D goodness-of-fit test for each of the distributions used. The improvements were approximately at the level of 0.01%.

Using the mean rank plotting position also improved the power of the A-D goodness-of-fit test for the uniform, the double exponential, and the Weibull distributions. The power remained almost the same for the exponential and the lognormal distributions. The improvements for the uniform, the double exponential, and the Weibull distributions were 0.01%.

The following table shows the approximate power improvements of the modified tests over the unmodified tests.

Table 5.1. Power Improvements of Modified Tests over Unmodified Tests

	A - D		*C - VM*	
	Median	Mean	Median	Mean
Uniform	.01%	.01%	5.0%	9.0%
Weibull	.01%	.01%	10.0%	2.0%
Exponential	.01%	No change	10.0%	Reduced
Lognormal	.01%	No change	10.0%	Reduced
Double Exp.	.01%	.01%	10.0%	Reduced

5.3 Conclusion

Both plotting positions improved the powers of the known A-D and C-VM goodness-of-fit tests. The median rank plotting position seemed to have better powers than the mean rank plotting position did.

VI. Conclusions and Recommendations

6.1 Conclusions

Based on the results obtained in this thesis. the following conclusions are noted:

1. The power comparison study based on the alternative distributions showed that both plotting positions increased the power of the Anderson-Darling goodness-of-fit test for almost all the distributions. The power remained the same only for the exponential and the lognormal distributions when the mean rank plotting position was used. The improvement for the C-VM goodness-of-fit test was considerably higher for the uniform and the Weibull distributions when either the median or the mean rank plotting position was used and. for the lognormal. the exponential. the Weibull. and the double exponential distributions when the median rank plotting position was used. The median rank plotting position showed a higher power than the mean rank plotting position.

2. The critical value tables for both tests were presented. The tables can be used to test whether a random sample of data follows the normal distribution.

6.2 Recommendations

Based on the observations made throughout the thesis study. the following research areas are proposed for further studies:

1. Apply the goodness-of-fit techniques developed in this thesis for other distribution functions.

2. Further modify the goodness-of-fit techniques developed in this thesis to allow for unknown values of the parameters of the hypothesized normal distribution.

```
C****************************************************************
C*                                                              *
C*     THIS PROGRAM DETERMINES THE CRITICAL VALUES WITH         *
C*  THE MODIFIED ANDERSON-DARLING AND CRAMER-VON MISES STATISTICS *
C*         VALUES FOR THE NORMAL DISTRIBUTION.                  *
C*                                                              *
C*                                                              *
C****************************************************************
C
C
C     ** SOME COMMENTS ABOUT THE VARIABLES USED **
C
C     S = Number of repetitions
C     N = Sample size
C     ANDER(*) = A-D Statistic values
C     CRIT(*) = Critical values at (*)
C     SEED = Input/Output double precision variable assigned an integer
C            value in the exclusive range
C     1,N (GGNO) = "1" is the first order statistic to be generated,
C     "N" is the last order statistic to be generated
C     XX(*) = Output vector of length [N+1-IFIRST] containing the order
C             statistics in an ascending order
C     IER = Error parameter output
C     ZZ = Input value at which function is to evaluated
C     YY = Output probability that a random variable having a normal
C          (0,1) distribution will be less than or egual to ZZ.
      INTEGER S,J,N,K,SS
      REAL XX(100),ZZ,TT,ANDER(0:5010),DARL
      REAL CRIT80,CRIT85,CRIT90,CRIT95,CRIT99
C
C     Enter the double precision seed
C
      DOUBLE PRECISION SEED
      SEED= 469857936.D0
C
C     Number of repetition is 5000
C
      S=5000
C
C     Enter the number of sample size
C
      N=80
```

```
      DO 11 J=1,S
C
C     Call GGNO to generate set of order statistics
C     from the normal distribution
C
          CALL GGNO(SEED,1,N,N,XX,ERR)
C
C     Call STANDAR to standardize the random deviates
C     by Equation(3.12).  The following computations will use
C     these stadardize values.
C
          CALL STANDAR(XX,N)
C
C     Call VSRTA to sort the arrays by algebraic values
C
          CALL VSRTA(XX,N)
           DO 22 K=1,N
            ZZ=XX(K)
C
C     Call MDNOR to determine the CDF values of the normal
C     distribution for the standardized data
C
            CALL MDNOR(ZZ,TT)
             XX(K)=TT
22        CONTINUE
C
C     Call ANDERSON to find the A-D statistic values
C
C     **Call CRAMER to find the CV-M statistic values
C
           CALL ANDERSON(N,XX,DARL)
C
C       ** CALL CRAMER(N,XX,DARL) **
C
           ANDER(J)=DARL
11      CONTINUE
         ANDER(0)=0.0
         SS= S+1
C
C     Call VSRTA to sort the statistics to determine the percentiles
C     of the array in order to find the critical values
C
         CALL VSRTA(ANDER,SS)
C
C     Call EXTRA to extrapolate the data for the points at "0",
```

```
C     and "n+1".  Output vector is an array of length "n+2".
C
         CALL EXTRA(S,ANDER)
C
C     Call VALUES to determine the critical values
C
        CALL VALUES(ANDER,CRIT80,CRIT85,CRIT90,CRIT95,CRIT99,S)
C
C     Print the critical values
C
      WRITE(*,*) CRIT80,CRIT85,CRIT90,CRIT95,CRIT99
      END


C
C     Using the technique explained in Chapter 3, find the critical values
C
      SUBROUTINE CAN(Y1,Y2,D1,D2,Y,RES)
      REAL M,B,Y1,D1,Y2,D2,Y,RES
      M= (Y2-Y1)/(D2-D1)
      B = Y1 - (M*D1)
      RES = (Y-B)/M
      END

C
C     The following subroutine calculates the A-D statistic values
C

      SUBROUTINE ANDERSON(K,R,ANDER)
      INTEGER K,I
      REAL R(*),TOTAL,XX,YY,ANDER,ZZ,RR
      TOTAL = 0.0
      DO 11 I=1,K
        RR = R(I)
         IF (RR . LT. 0.0 .AND. RR.EQ.0.0) THEN
            RR = .0001
         ENDIF
        ZZ = 1.0 - R(K-I+1)
        IF(ZZ.LE.0.0)THEN
             ZZ = .0001
        ENDIF
        XX = LOG(RR)
        YY = LOG(ZZ)
        TOTAL = (((2.0*REAL(I))-0.635)*(XX+YY))+TOTAL
11    CONTINUE
```

```
      ANDER = - REAL(K) - ((1.0/(REAL(K)-0.365))*TOTAL)
      END

C
C    The following subroutine calculates the CV-M statistic values
C

      SUBROUTINE CRAMER(N,R,ANDER)
      INTEGER I,N
      REAL R(*),TOTAL,ANDER,MEDIAN
      TOTAL =0.0
      DO 22 I=1,N
      MEDIAN = (REAL(I)-0.3175)/(REAL(N)-0.365)
       TOTAL = TOTAL + ((R(I) - MEDIAN) * (R(I) - MEDIAN))
22    CONTINUE
      ANDER = (1.0 / (12.0 * REAL(N))) + TOTAL
      END

C
C     Standardize all the data
C

      SUBROUTINE STANDAR(X,N)
      INTEGER I,N
      REAL X(*),XSUM,XBAR,S,XOUT
      XSUM = 0.0
      XOUT = 0.0
      DO 100 I = 1,N
        XSUM =  XSUM + X(I)
100   CONTINUE
C
C     Compute the mean
C
      XBAR=XSUM/N
      DO 200 I =1,N
        XOUT = XOUT+((X(I)-XBAR)*(X(I)-XBAR))
200   CONTINUE
      S = SQRT(XOUT/(N-1))
      DO 300 I = 1,N
C
C     Compute the standardized data
C
        X(I) = (X(I)-XBAR)/S
300   CONTINUE
      END
```

```
      SUBROUTINE EXTRA(N,D)
      INTEGER N,NO,N1
      REAL Y1,Y2,D(0:*),D1,D2,ZZ
      Y1 = 0.5/N
      Y2 = 1.5/N
      D1 = D(1)
      D2 = D(2)
      CALL CAN(Y1,Y2,D1,D2,0.0,ZZ)
      IF(ZZ.GE.0.0) THEN
         D(0) = ZZ
        ELSE
         D(0) = 0.0
      ENDIF
      Y1 = (REAL(N) -1.5)/N
      Y2 = (REAL(N) -0.5)/N
      NO = N-1
      D1 = D(NO)
      D2 = D(N)
      CALL CAN(Y1,Y2,D1,D2,1.0,ZZ)
      N1 = N+1
      D(N1) = ZZ
      END

C
C     The following subroutine determines the percentiles
C     and finds the critical values by evoking
C     the subroutine CAN
C


      SUBROUTINE VALUES(D,CRIT80,CRIT85,CRIT90,CRIT95,CRIT99,N)
      INTEGER I,N,NN
      REAL D(0:*),Y(0:6000),C80,C90,C95,C99,C85,
     1   Y79,D79,Y81,D81,DIF90,Y89,Y91,D89,D91,DIF95,DIF80,
     1   Y94,Y96,D94,D96,DIF99,Y98,Y100,D98,D100,DIF85,
     1   Y84,D84,Y86,D86,CRIT85,CRIT80,CRIT90,CRIT95,CRIT99
      DO  100  I=1,N
        Y(I) = (REAL(I) - 0.5)/REAL(N)
100   CONTINUE
```

```
       Y(0) = 0.0
      NN = N + 1
       Y(NN) = 1.0
      C80 = 1000.0
      C85 = 1000.0
      C90 = 1000.0
      C95 = 1000.0
      C99 = 1000.0
      DO  200  I = NN,0,-1
         IF (Y(I).LE.0.75) GO TO 300
         IF (Y(I).GT.0.75 .AND. Y(I).LE.0.80) THEN
C
C     Get the desired percentile at 80%
C
           DIF80 = .80 - Y(I)
           IF (DIF80.LE.C80) THEN
             C80 = DIF80
             Y79 = Y(I)
             D79 = D(I)
             Y81 = Y(I+1)
             D81 = D(I+1)
           ENDIF
         ELSEIF (Y(I).GT..80.AND.Y(I).LE..85) THEN
C
C      Get the desired percentile at 85%
C
             DIF85 = .85 - Y(I)
             IF (DIF85.LE.C85) THEN
                C85 = DIF85
                Y84 = Y(I)
                D84 = D(I)
                Y86 = Y(I+1)
                D86 = D(I+1)
              ENDIF
          ELSEIF (Y(I).GT..85.AND.Y(I).LE..90) THEN
C
C      Get the desired percentile at 90%
C
                DIF90 = .90 - Y(I)
                IF (DIF90.LE C90) THEN
                  C90 = DIF90
                  Y89 = Y(I)
                  D89 = D(I)
                  Y91 = Y(I+1)
                  D91 = D(I+1)
```

```
                ENDIF
           ELSEIF (Y(I).GT..90.AND.Y(I).LE..95) THEN
C
C     Get the desired percentile at 95%
C
                DIF95 = .95 - Y(I)
                IF (DIF95 .LE. C95) THEN
                   C95 = DIF95
                   Y94= Y(I)
                   Y96= Y(I+1)
                   D94= D(I)
                   D96 = D(I+1)
                 ENDIF
            ELSEIF (Y(I).GT..95.AND.Y(I).LE..99) THEN
C
C     Get the desired percentile at 99%
C
                 DIF99 = .99 - Y(I)
                 IF (DIF99 .LE.C99) THEN
                    C99 = DIF99
                    Y98 = Y(I)
                    Y100 = Y(I+1)
                    D98 = D(I)
                    D100 = D(I+1)
                 ENDIF
            ENDIF
200   CONTINUE
300   CONTINUE
      IF (DIF80.EQ.0.0) THEN
         CRIT80 = D79
        ELSE
C
C    Compute the critical value at the significance level of .20
C
         CALL CAN(Y79,Y81,D79,D81,.80,CRIT80)
      ENDIF
      IF (DIF85.EQ.0.0) THEN
         CRIT85 = D84
        ELSE
C
C    Compute the critical value at the significance level of .15
C
          CALL CAN(Y84,Y86,D84,D86,.85,CRIT85)
      ENDIF
      IF (DIF90.EQ.0.0) THEN
```

```
           CRIT90 = D89
         ELSE
C
C    Compute the critical value at the significance level of .10
C
           CALL CAN(Y89,Y91,D89,D91,.90,CRIT90)
       ENDIF
       IF (DIF95.EQ.0.0) THEN
           CRIT95 = D94
         ELSE
C
C    Compute the critical value at the significance level of .05
C
            CALL CAN(Y94,Y96,D94,D96,.95,CRIT95)
       ENDIF
       IF (DIF99 .EQ.0.0) THEN
           CRIT99 = D98
         ELSE
C
C    Compute the critical value at the significance level of .01
C
            CALL CAN(Y98,Y100,D98,D100,.99,CRIT99)
       ENDIF
       END
```

```
C******************************************************
C*                                                    *
C*     THIS PROGRAM EXECUTES THE POWER COMPARISON     *
C*  FOR THE ANDERSON-DARLING AND THE CRAMER-VON MISES *
C*  TESTS WITH THE UNIFORM DISTRIBUTION AT THE GIVEN  *
C*                    SAMPLE SIZE                     *
C*                                                    *
C******************************************************
C
C
C     ***** SOME COMMENTS ABOUT THE VARIABLES USED *****
C
C     N = Sample size used
C     POWER(*) = The rejection number at the given level
C     SEED = Input/Output double precision variable assigned
C            an integer value in the exclusive range
C     ANDAR, AND = The known and the modified A-D statistic values
C                  calculated
C     CRAMER1, CRAMER2 = The known and the modified CV-M statistic
C                        values calculated
C
      INTEGER N,J,K,L,M,COUNT(4),POWER(30),CNT1,CNT2,I
      REAL WK(360),R(120),S(120),T(120),
     1     Y,P,ANDAR,AND,CRAMER1,CRAMER2,PWR(30)
      DOUBLE PRECISION SEED1
C
C     Enter the double precision seed
C
      SEED=1095785.D0
      OPEN(UNIT=1,FILE='UP80.OUT',STATUS='UNKNOWN')
      DO 600 I=1,30
          POWER(I)=0
600   CONTINUE
C
C     Enter the sample size
C
      N=80
      DO 100 J=1,5000
C
C     Generate the Uniform deviates
C
          CALL GGUBS(SEED1,N.R)
C
C     Sort the deviates
C
```

```
         CALL VSRTA(R,N)
      DO 200 K=1,N
        S(K) = R(K)
200   CONTINUE
C
C     Standardize the data
C
      CALL ESTPAR(S,N)
C
C     Sort the standardized data
C
      CALL VSRTA(S,N)
      DO 400 L=1,N
         Y=S(L)
C
C     Determine the CDF values of the uniform distribution
C     for the standardized data
C
         CALL MDNOR(Y,P)
         S(L)=P
400   CONTINUE
C
C     Find the statistic values for the following tests
C
      CALL ANDAR1(N,S,ANDAR)
      CALL ANDAR2(N,S,AND)
      CALL CVM1(N,S,CRAMER1)
      CALL CVM2(N,S,CRAMER2)
C
C     Enter the critical values for the sample size 80 from
C     the critical value tables presented in Chapter 4
C
      IF (ANDAR.GT.0.5018578) THEN
         POWER(1)=POWER(1)+1
      ENDIF
      IF (ANDAR.GT.0.5535432) THEN
         POWER(2)=POWER(2)+1
      ENDIF
      IF (ANDAR.GT.0.6213608) THEN
         POWER(3)=POWER(3)+1
      ENDIF
      IF (ANDAR.GT.0.7532310) THEN
         POWER(4)=POWER(4)+1
      ENDIF
      IF (ANDAR.GT.1.046329) THEN
```

```
   POWER(5) =POWER(5)+1
ENDIF
IF (AND.GT. 1.602745) THEN
   POWER(6)=POWER(6)+1
ENDIF
IF (AND.GT. 1.650009)THEN
   POWER(7)=POWER(7)+1
ENDIF
IF (AND.GT. 1.714119)THEN
   POWER(8)=POWER(8)+1
ENDIF
IF(AND.GT. 1.824196)THEN
   POWER(9)=POWER(9)+1
ENDIF
IF(AND.GT. 2.109204)THEN
   POWER(10)=POWER(10)+1
ENDIF
IF(CRAMER1.GT.0.079321444)THEN
   POWER(11)=POWER(11)+1
ENDIF
IF(CRAMER1.GT.0.089231446)THEN
   POWER(12)=POWER(12)+1
ENDIF
IF(CRAMER1.GT.0.1024222)THEN
   POWER(13)=POWER(13)+1
ENDIF
IF(CRAMER1.GT.0.1257416)THEN
   POWER(14)=POWER(14)+1
ENDIF
IF(CRAMER1.GT.0.1837831)THEN
   POWER(15)=POWER(15)+1
ENDIF
IF(CRAMER2.GT.0.083445959)THEN
   POWER(16)=POWER(16)+1
ENDIF
IF(CRAMER2.GT.0.093562253)THEN
   POWER(17)=POWER(17)+1
ENDIF
IF(CRAMER2.GT.0.1065563)THEN
   POWER(18)=POWER(18)+1
ENDIF
IF(CRAMER2.GT.0.1313581)THEN
   POWER(19)=POWER(19)+1
ENDIF
IF(CRAMER2.GT.0.1670539)THEN
```

```
          POWER(20)=POWER(20)+1
        ENDIF
100     CONTINUE

         DO 500 I=1,20
C
C        Determine the rejection number
C
           PWR(I)=POWER(I)/5000.0
500      CONTINUE
         WRITE(1,*)'FOR N =' , N , 'POPULATION = UNIFORM'
         DO 11 I=1,20
C
C        Print the rejection number
C
         WRITE(1,*) PWR(I)
11       CONTINUE
         END

C
C        The subroutines are presented in the previous section
C
C
```

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